# Problem of Electronic-Optical System Synthesis for High Power Electron Gun

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Abstract - The computation of electron gun focusing electrodes is usually based upon the method of synthesis. It is usually assumed that electrodes boundaries coincide with the electron-beam. The situation becomes completely different during the generation of high power electron flows (for example, in multi-gap electron guns). Here the formal "extension" of electrodes along the equipotential curves in radial direction changes electric field at the bound of the beam and this in it's turn changes the flow configuration. The spatial charge is also has to be taken into the consideration. To calculate such beams, we have to modify the synthesis problem as at the part of the problem definition, as at the part of the problem solution. The paper is dedicated to analysis of difficulties appearing during such modification.

#### Introduction

The computation of electron gun focusing electrodes is usually based upon the method of synthesis. This method includes two problems: inner and outer. The distribution of electric and hydrodynamic parameters of the flow (density, potential, diameter, etc.) is determined in a frame of the inner problem. The calculation of the electric fields outside the flow is carried out in a frame of outer problem. The calculated fields allow one to determine the form of accelerating electrodes and their potential, required to provide the motion of the charged particles flow calculated in the first problem. Mathematically it looks like solution of the Cauchi problem for the Laplace's equation in the area outer with respect to the beam. The spatial potential distribution is determined by given potential distribution and its normal derivative at the beam border.

This approach is justified if the electrodes side with the beam or are submerged into it fully or partially. They define the potential at the beam bound independently on spatial charge of the flow. Similar schemes can be realized in guns with guiding nets, low power klystrons, progressing wave lamps and other devices of SHF-electronics where power dissipated on the electrodes is too low for their destruction.

The situation becomes completely different during the generation of high power electron flows (for example, in multi-gap electron guns). The apertures of holes in forming electrons are several times larger than the beam diameter. The formal "extension" of electrodes along the equipotential curves in radial direction changes electric field at the bound of the beam and this in it's turn changes the flow configuration. This happens because the equipotential curves are solution of OUTER problem.

The problem of electrodes form definition in multi-gap gun with electrode holes with diameters essentially larger than the e-beam is considered in this paper. The computation method considering the spatial e-beam charge is proposed. The additional questions appearing during the solution are analyzed.

## The Peculiarity of Outer Cauchi Problem

Let us consider the peculiarities of outer Cauchi problem by the example of two-dimensional area I, confined by the contour  $\alpha$  and surrounded by the area II with boundary  $\beta$  (Fig. 1). The definition of the potential distribution  $\varphi_{\beta}$  on  $\beta$  contour uniquely defines potentials in I and II (as the solution of the inner Dirichlet's problem). The potential distributions  $\varphi_{\alpha 1}$ ,  $\varphi_{\alpha 2}$  and  $\varphi_{\alpha 3}$  on contours  $\alpha$ ,  $\alpha_1$  and  $\alpha_2$  are also determined ununiquely. At the same time the inner Dirichlet's problem solution is the same in area I for each of these contours because of its uniqueness.



Fig. 1. Two-dimensional areas I and II

The solution in the area I would be invariable if the appropriate potentials are set at equipotential curves a, b, c, d, e (Fig. 1). If area I is screened from contour  $\beta$  by these electrodes then contour  $\beta$  could be eliminated – because the potential at the area would not "feel" it. The solution would not change if the equipotential curves a, b, ..., e would be partially filled with electrodes under the stipulation that screening is kept unchanged. So moving away of electrodes to contour  $\alpha_1$  defines the potential  $\phi_{\alpha 1}$  distribution and the solution doesn't change in I (Fig. 2).



Fig. 2. Contour  $\beta$  elimination

In case of electron gun this means, that moving away of electrodes would not change the potential at the e-beam area if they continue to coincide with equipotential curves. These curves are the solution of INNER problem for some area containing and screening area I. The solution of OUTER problem is quite a different task. Let us set the potential  $\varphi_{\alpha}$  distribution at the contour  $\alpha$  and  $f_{\beta} \neq \phi_{\beta}$  at the contour  $\beta$ . These boundary conditions do not contradict each other if one picture area II as inner area confined by  $\alpha$ and  $\beta$  contours and infinitely near lines  $\gamma_1$  and  $\gamma_2$ (Fig. 3). The solution would remain the same in area I, but would change in area II. The potential distribution at contours  $\alpha_1$  and  $\alpha_2$  would become  $f_1$  and  $f_2$ , which are not equal to  $\varphi_{\alpha 1}$  and  $\varphi_{\alpha 2}$ . The forms of equipotential curves would also change at the area II.



Fig. 3. The area II presentation as an inner area

Now, if we want to keep the solution at the area I by the elimination of contour  $\beta$  we would have to align the electrodes with equipotential curves in both I and II areas or at least at area II (Fig. 4). In latter case the electrodes have to be located near the contour  $\alpha$  defining  $\varphi_{\alpha}$  distribution at the boundary  $\alpha$ . If the electrodes were moved away from the  $\alpha$  contour, for exam-

ple to contour  $\alpha_1$ , then they would define distribution  $f_1$  (instead of  $\phi_{\alpha 1}$ ). And the solution will change in area I.



Fig. 4. Potential set by the help of electrodes coinciding with equipotential curves

This means that moving away of electrodes along the equipotential curves obtained as a solution of the OUTER problem would cause electric field to change in the electron gun. To avoid this we have to obtain equipotential curves as solution of the INNER problem.

The definition of the potential on some rotation figure surface A (Fig. 5) unambiguously defines the distribution of the potential in the inner area I in case of axially symmetric geometry. At the same time additional conditions are required to define the potential distribution at the outer area:

- either the potential definition at the rotation surface B (so the solution between A and B would be known),

- either the potential derivative definition (for example normal to the surface) at A (so the solution at the area II which is outer in relation to A would be known).



Fig. 5. A and D totation surfaces

In compliance with written above we are interested in the solution in area II. This solution corresponds to the inner problem. In other words, the potential distribution on the B surface should be such that its solution would coincide with one obtained at I with the same potential distribution at A. In this case the conditions on B should be unambiguously defined by the conditions on A. The solution at A appears as inner solution for area confined by the B surface.

A method would be suggested below for solution of following problem: how to find the potential distribution on surface B that does not change the solution in the area I by the defined potential distribution on the surface A.

### The Influence of the E-Beam Space Discharge and the Mechanism of Synthesis Problem Solution

The second problem appears during the moving away of the electrodes from the e-beam. It is caused by the fact that the space charge of e-beam plays considerable role in the creation of the potential at the e-beam boundary. This means, that potential distribution at the e-beam boundary obtained during the solution of the inner problem of charged flow synthesis should be amended. The amount of amend is defined by the space charge of the e-beam. The electrodes form and their potentials should be defined by amended potential distribution.

This circumstance complicates the process of electrodes calculation at first glance. But method proposed in this paper allows one avoiding following complications: first – connected with finding of the potential along with e-beam boundary; second – connected with following spatial potential restoration. This method allows obtaining simple and vivid solutions in some cases. We have to note that we talk only about solution of the OUTER synthesis problem. Inner problem is solved by the traditional methods [1, 2]. The essence of the method is reduced to following.

The e-beam boundary, its potential and spatial charge density distribution are defined by the results of inner synthesis problem solution. The necessity of potential derivative normal to the e-beam boundary is no longer relevant. Then the potential at the e-beam boundary generated by the elimination of the spatial charge is defined. The inner Dirichlet's problem is solved at the next stage by the known potential distribution at the e-beam boundary. The potential distribution at the system symmetry axis is also defined. And, at last, the spatial potential distribution and equipotential curves form, i.e. electrodes potential and configuration is restored by the known axial distribution. The method can be modified if the potential at the symmetry axis and charge density becomes known during the inner problem solution. In this case the axial potential distribution can be calculated even simpler, as difference between the axial potential in the beam and potential generated by the space charge.

As one can see in contrast to common methods several simplifications occur: first – the normal potential derivative at the e-beam boundary does not applied, because the inner Dirichlet's problem is solved; second – the necessity of the solution of Cauchi's problem for the Laplace's equation for the area with curved boundaries is eliminated. The latter problem can be far from usually solved as it shown in [3], because of lack of well-conditioned straightening transformations.

But the problem of restoration of spatial potential distribution by the axial potential distribution appears in the proposed method. This problem is ill conditioned [4]. Its solution is possible by the method of analytical continuation [5, 6] if the precise axial potential distribution is known. It can be also solved with help of regularizing algorithms described in [7].

So, the proposed method can be used for solution of outer synthesis problem for electron optical system of high-power electron gun with electrodes apertures significantly larger then the beam diameter.

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