

# Comparative Analysis of Three Power Amplification Schemes for Multi-MA Drivers Based on Microsecond Inductive Energy Storage<sup>1</sup>

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**Abstract** – During the past twenty years, extensive studies were conducted to sharpen current pulse on IES (Inductive Energy Storage) drivers using the well-known Plasma Opening Switch (POS) scheme. Due to the emergence of new power amplification concepts, Flux Compression (FC), LL scheme (Chuvatin), Current Doubler Flux Compression (Rudakov), etc..., new opportunities appear for improving IES generators operation. In order to evaluate and compare the performances of three of these schemes (POS, LL and FC), this paper proposes a system analysis based on a 0D modeling. The system configuration is common for the three schemes: a RLC generator – the power amplification scheme studied – a Z-pinch load. For each scheme, the energetic efficiency is presented and analyzed.

## 1. Introduction

In studying power amplification part for its multi-mega amperes IES generator, CEG had opportunity to test several very different schemes that could realize this function. Among them, CEG has tested a composite plasma opening switch [1], the LL scheme [2] and the flux compression scheme [3]. The POS are studied for a long time [4, 5] and the novelty was here to separate the conditions for the conduction phase from those of the opening phase in order to increase the performances (opening time and maximum opening resistance). The LL scheme is a recent idea of A. Chuvatin. The principle is to generate a voltage using a rapidly moving plasma traveled through by a high current ( $\vec{v} \wedge \vec{B}$  term of the generalized Ohm's law). The proof of principle was demonstrated on GIT12 at HCEI [6]. The flux compression scheme comes directly from the explosive generators principles where the solid liner was replaced by a plasma one and it is imploded with a high magnetic field generated by a high current. This scheme was tested in the 100 nanosecond regime [3] and in the microsecond regime [7, 8].

All these three schemes were tested on an IES generator with a quarter period around 1  $\mu$ s and a current lying between 3 and 5 MA. In order to evaluate and compare the efficiency of these three different schemes, we have first defined a common framework

and then we have modeled analytically the transfer of energy for each of these schemes.

## 2. Definition of the Common Framework

For each scheme, we will study a configuration composed by a generator connected to the studied power amplification scheme and a z-pinch as load. The equivalent electrical scheme is shown in Fig. 1.

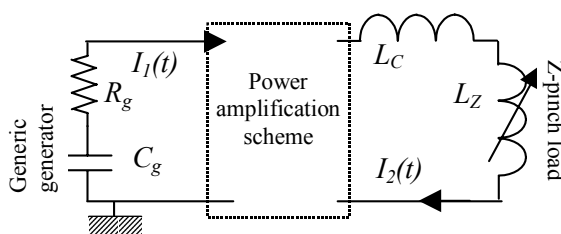


Fig. 1. Generic configuration for power amplification scheme study

Here  $C_g$  and  $R_g$  are the capacitance and the resistance of the generator,  $L_C$  and  $L_Z$  are the connection and the load inductances. We will call the energy stored in the  $L_X$  inductance as  $E_X$  ( $X = Z, S, \dots$ ) and as  $E_c$  the kinetic energy of the pinch. In order to simplify the analysis, the scheme operation will be separated into three phases. The first one, from 0 to  $t_c$ , is the transfer of the energy of the generator to the power amplification scheme. During this phase, the inductance of the scheme,  $L_S$ , seen by the generator, will be considered as constant. The second phase, between  $t_c$  and  $t_o$ , is the switching of the energy from the scheme to the load. During this phase the generator will be considered as a short-circuit and the load will have a constant inductance. And the third phase, between  $t_o$  and  $t_r$ , will be the compression of the z-pinch. During this phase, the state of the scheme (inductance, resistance, ...), seen by the load, will be constant. This description is suitable for the POS and LL schemes. For the FC scheme the first two steps happen together. For each phase, we can define an energy efficiency as

$$\eta_1 = E_S(t_c)/E_{Cg}, \eta_2 = E_S(t_o)/E_S(t_c), \eta_3 = E_c(t_r)/E_Z(t_r).$$

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So, the total energy efficiency for a scheme is defined as the ratio of the kinetic energy of the z-pinch load over the initial stored energy:

$$\eta_{Scheme} = \frac{E_c(t_r)}{E_{Cg}} = \eta_1 \eta_2 \frac{E_Z(t_r)}{E_Z(t_o)} \eta_3.$$

In the calculations, it is convenient to define the current efficiency as  $k_2 = I_2(t_o)/I_1(t_o)$  and  $k_3 = I_2(t_r)/I_2(t_o)$ .

### 3. Compression Phase

During this phase the inductance of the z-pinch load vary from  $L_{Zo}$  to  $L_{Zo} + \Delta L_Z$ . The energy supplied to the load can be expressed in the following form:

$$E_Z(t_r) = \frac{(L_{Zo} + \Delta L_Z)I_2^2(t_r) - L_{Zo}I_2^2(t_o)}{2} + \int_{t_o}^{t_r} \frac{I_2^2}{2} \frac{dL_Z}{dt} dt.$$

The first term correspond to the magnetic energy stored around the pinch and the second one is the kinetic energy that is typical of the total radiated energy. So the efficiency

$$\eta_3 = 1 - I_2^2(t_o) \frac{(L_{Zo} + \Delta L_Z)k_3^2 - L_{Zo}}{2E_Z(t_r)}$$

Now we have to express  $E_Z(t_r)$  from the state of each scheme at the end of the switching phase.

### 4. Efficiency of the POS Scheme

The POS scheme can be modeled as shown in Fig. 2.

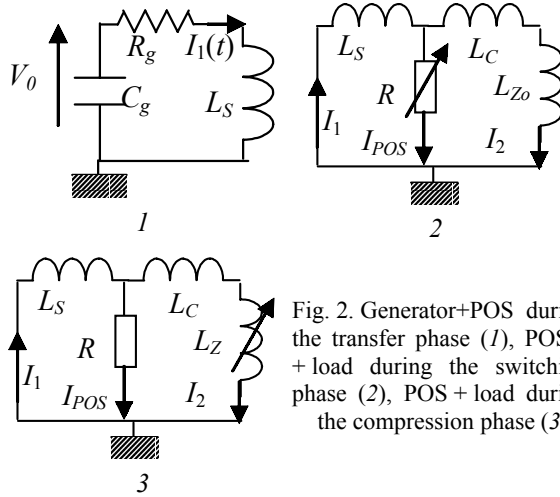


Fig. 2. Generator+POS during the transfer phase (1), POS + load during the switching phase (2), POS + load during the compression phase (3)

The first phase is the transfer of the energy from the capacitance into the inductance. It has the well known expression for the pseudo-periodic regime

$$\eta_1 = \frac{L_{So} I_1^2(t_c)}{C_g V_o^2} \approx \exp(-2\alpha t_c) \sin^2(\omega_o t_c)$$

with  $\omega_0 = 1/\sqrt{L_{So} C_g}$  for the natural system frequency and  $2\alpha = R_g/L_S$  for the damping factor. Typically,

$\eta_1 \approx 0.8$  for our generators if  $t_c$  is the quarter period. Solving the system of coupled equations for the switching phase, we obtain the expression of  $\eta_2$ :

$$\eta_2 = \frac{L_S L_Z}{(L_S + L_C + L_{Zo})^2} \left( 1 - \exp\left(-\frac{1}{L_{eq}} \int_{t_c}^{t_o} R(t) dt\right) \right)^2$$

with

$$L_{eq} = \frac{L_S(L_C + L_{Zo})}{L_S + L_C + L_{Zo}}.$$

In order to evaluate the efficiency during the compression phase, it is necessary to do a strong hypothesis on the value of the resistance. We will consider the case (1), where  $R(t_o) = 0$  and the case (2), where  $R(t_o)$  is high enough to be considered as an open circuit. The first case corresponds to the frequently observed reclosure of the POS and the second one corresponds to the ideal case. Finally, the total efficiency of the system in the two cases is:

case (1)

$$\eta_{POS} = \frac{\eta_1 L_S (L_C + L_{Zo}) \Delta L_Z \left( 1 - \exp\left(-\frac{1}{L_{eq}} \int_{t_c}^{t_o} R(t) dt\right) \right)^2}{(L_S + L_C + L_{Zo})^2 (L_C + L_{Zo} + \Delta L_Z)},$$

case (2)

$$\eta_{POS} = \frac{\eta_1 L_S \Delta L_Z \left( 1 - \exp\left(-\frac{1}{L_{eq}} \int_{t_c}^{t_o} R(t) dt\right) \right)^2}{(L_S + L_C + L_{Zo})(L_S + L_C + L_{Zo} + \Delta L_Z)}.$$

To simplify the study, we focus on the first case and we use dimensionless variables. Moreover, according to experimental observations, we suppose that the variation of the resistance is triangular with the maximum resistance  $R_o$  and the time base  $\Delta t$ . So we can now define the dimensionless variables  $x = (L_C + L_{Zo})/L_S$ ,  $y = (R_o \Delta t)/L_S$ , and  $p = \Delta L_Z/L_S$  and the expression becomes:

$$\eta_{POS} = \eta_1 \frac{xp \left( 1 - \exp\left(-y \frac{x+1}{2x}\right) \right)^2}{(1+x)^2 (x+p)}$$

The argument of the exponential term takes the form  $(\Delta t/\tau_G + \Delta t/\tau_Z)$ , where  $\tau_G$  and  $\tau_Z$  are the characteristic response times of the different parts of the circuit. For  $\eta_1 = 0.8$  and  $p = 0.25$  this relation is presented in Fig. 3.

We can see an optimum area that is located at  $x = 0.3$  when the exponential term is negligible. The optima are more or less easy to find depending on the dimensionless variables we choose. **The maximum energy efficiency expected in this configuration stands between 6 and 7%. In the ideal case, this**

**efficiency goes up till 11%.** It is now interesting to compare this analytical formula with the experimental results. All the experiments conducted at CEG were done with a fixed load inductance, and so we have adapted the upper formula to that case. We have calculated all the geometric inductances, and the resistance was estimated from voltage and current measurements [9, 10]. The experimental results belong to 2 different series of experiments on 2 different generators and the configuration in each series was quite different (1, 2 or 3 rows of plasma guns injection). So, for comparison, we will consider only the switching phase energy efficiency  $\eta_2$ . The results are shown in Fig. 4.

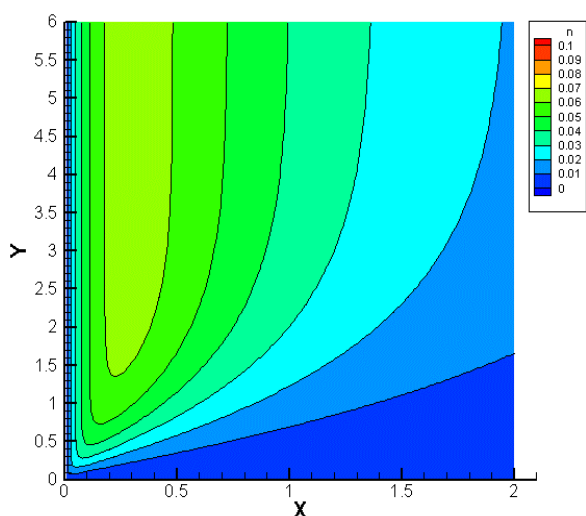


Fig. 3. Energy efficiency of the POS scheme

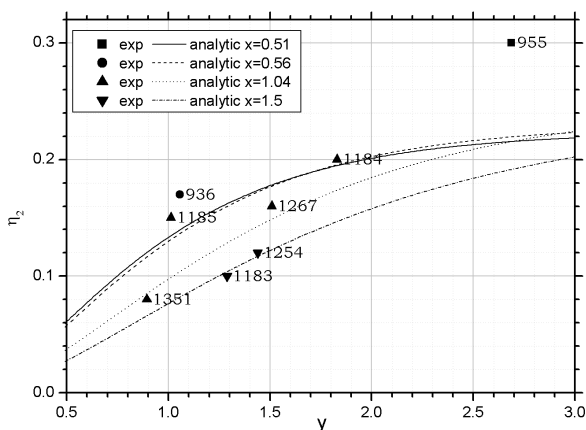


Fig. 4. Comparison of experimental and analytical results for  $\eta_2$

This comparison shows that the analytical formula allows us to predict the energy efficiency of the POS scheme with an error less than 50%.

## 5. Efficiency of the LL Scheme

The LL scheme can be modeled as shown in Fig. 5.

For the first phase, the energy efficiency is exactly the same as for the POS case. During the switching phase, the variable inductance goes from  $L_{A0}$  to

$L_{A0} + \Delta L_A$ . Solving the system of coupled equations for this phase leads to the expression

$$\eta_2 = \frac{L_{Z0} L_S (\Delta L_A)^2}{((L_{A0} + \Delta L_A)(L_S + L_C + L_{Z0}) + L_S(L_C + L_{Z0}))^2}.$$

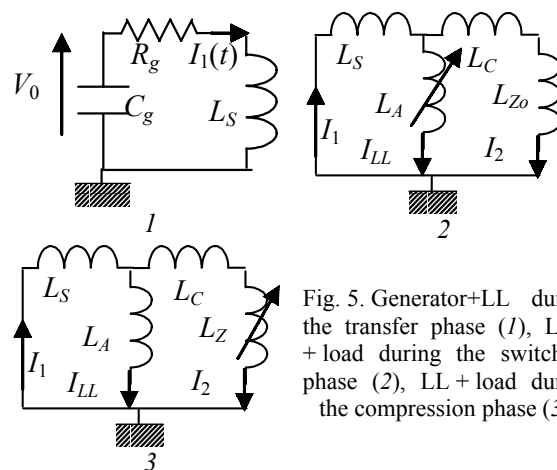


Fig. 5. Generator+LL during the transfer phase (1), LL + load during the switching phase (2), LL + load during the compression phase (3)

In order to evaluate the efficiency during the compression phase, it is necessary to do a strong hypothesis on the value of the inductance  $L_A$  at the end of the switching phase. We will consider the case (1), where  $L_A(t_0) = 0$  and the case (2), where  $L_A(t_0)$  is high enough to be considered as an open circuit. The first case corresponds to a failure of the scheme and the second one corresponds to the ideal case. Finally, the total efficiency of the system in the two cases is:

case (1)

$$\eta_{LL} = \frac{\eta_1 L_S (\Delta L_A)^2 (L_C + L_{Z0}) \Delta L_Z}{(L_C + L_{Z0} + \Delta L_Z)((L_{A0} + \Delta L_A)(L_S + L_C + L_{Z0}) + L_S(L_C + L_{Z0}))^2},$$

case (2)

$$\eta_{LL} = \frac{L_S (\Delta L_A)^2 ((L_S + L_C + L_{Z0}) - (L_S + L_C + L_{Z0} + \Delta L_Z) k_3^2)}{((L_{A0} + \Delta L_A)(L_S + L_C + L_{Z0}) + L_S(L_C + L_{Z0}))^2},$$

$$k_3 = \frac{(L_{A0} + \Delta L_A + L_S)(L_C + L_{Z0}) + L_S(L_{A0} + \Delta L_A)}{(L_{A0} + \Delta L_A + L_S)(L_C + L_{Z0} + \Delta L_Z) + L_S(L_{A0} + \Delta L_A)}.$$

We focus now on the second case and we use the same dimensionless variables  $x$  and  $p$  as those defined in the POS case. We need to define 2 more specific variables  $y = \Delta L_A / L_S$ , and  $z = L_{A0} / L_S$  and the expression becomes:

$$\eta_{LL} = \eta_1 \frac{y^2 ((1+x) - (1+x+p)k_3^2)}{((y+z)(1+x) + x)^2}.$$

For  $\eta_1 = 0.8$  and  $p = 0.25$  this relation is presented in Fig. 6 with the upper foreground quarter removed.

At  $z$  being fixed, there is an optimum  $y$  that depends on  $x$ . The optima are more or less easy to find depending on the dimensionless variables we choose. **The maximum energy efficiency that can be expected in this configuration lies between 8% and 14%. In the failure mode, this efficiency falls down to 5%.** This shows that LL scheme is very sensitive to failure mode, more than the POS scheme. All the experiments conducted on GIT12 driver at Tomsk were done with an open circuit as load. So the comparison between analytical estimations and experimental results could only be done with the help of modeling of the LL scheme in a circuit code, which is beyond the scope of this paper. This work was done in Ref. [11] and leads to uncertainty on the analytical estimation less than 40%.

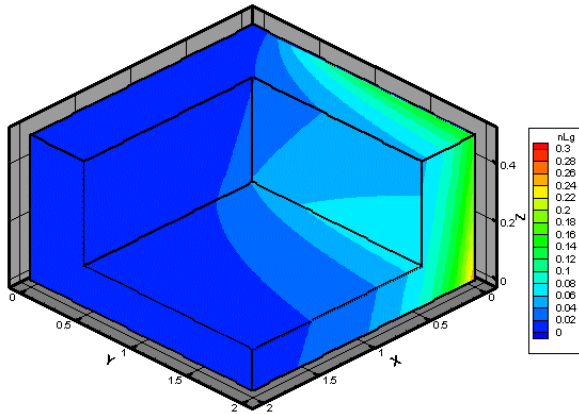


Fig. 6. Energy efficiency of LL scheme

## 6. Efficiency of the CF Scheme

For the CF scheme, transfer and switching phases happen together and the electrical circuit is modeled as shown in Fig. 7. The primary circuit is on the right and the secondary is on the left. The bolded arrow indicates that inductances vary such a manner that  $L_{S1} + L_{S2}$  is constant. So  $L_{S1} = L_{S1o} + \Delta L_S$  and  $L_{S2} = L_{S2o} - \Delta L_S$ .

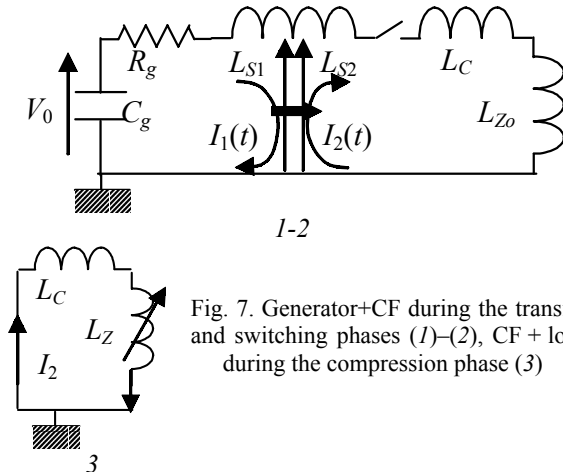


Fig. 7. Generator+CF during the transfer and switching phases (1)–(2), CF + load during the compression phase (3)

Doing the assumption that the kinetic energy of the liner is null when it arrives on the central rod, it is possible to express the total energy efficiency:

$$\eta_{CF} = \frac{\eta_1(t_o) L_{S1o} \Delta L_Z (L_{S2o} + L_C + L_{Zo})}{(L_{S1o} + \Delta L_S)^2 (L_{S2o} - \Delta L_S + L_C + L_{Zo} + \Delta L_Z)}$$

We can still use the dimensionless variables  $x$  and  $p$  with  $L_S$  replaced by  $L_{S1o}$ . Defining the specific variables  $y = \Delta L_S / L_{S1o}$  and  $z = L_{S2o} / L_{S1o}$ , the expression is

$$\eta_{CF} = \eta_1(t_o) \frac{p(z+x)}{(z-y+x+p)(1+y)^2}$$

Considering an ideal case ( $y = z$ ), this relation is presented in Fig. 8 for  $\eta_1(t_o) = 0.8$  and  $p = 0.25$ . The optimum area for low  $y$  corresponds to configurations that are not consistent with the use of a  $z$ -pinch load and, therefore, it must be avoided. **The maximum energy efficiency that we could hope with this scheme lies between 15% and 20%.** Nevertheless, this figure shows that there is a strong variation of the efficiency according to  $x$  coordinate. That means that the efficiency can fall down to 10% due to a small variation of the  $L_C$  inductance. Experimentally we have observed that the plasma liner thickness adds significant inductance to  $L_C$  and so the efficiency is decreased significantly. Taking into account this limitation, the analytical formula gives the efficiency with less than 20% uncertainty.

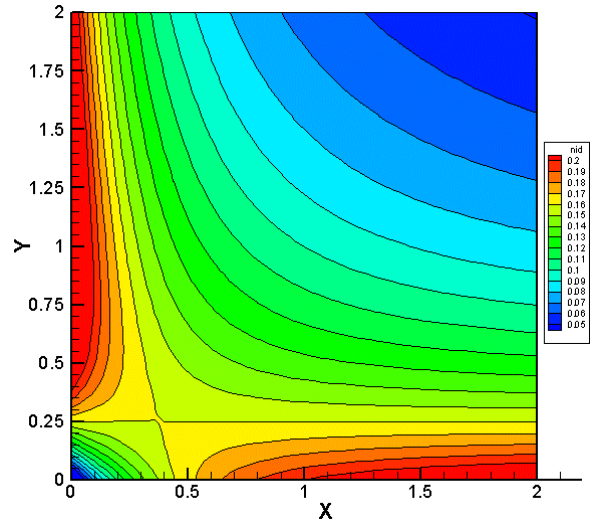


Fig. 8. Energy efficiency of CF scheme

## 7. Conclusion

The modeling presented in this paper allows to predict the energy efficiency with an error less than 50%. In an optimized case, this efficiency can reach around 11% for the POS, 14% for the LL scheme and 20% for the CF scheme. In the failure modes (plasma reclosure

for POS, upstream current shunting for LL and magnetic flux losses in plasma for CF), this efficiency falls down to 7, 5 and 10% respectively.

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