Models of the Plasma Corona Formation and Stratification of Exploding Micro-Wires

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Abstract – In the report models of plasma corona formation and stratification of a gas-plasma core of exploding micro-wires are offered and discussed.

1. Introduction

Nanosecond electric explosion of wires of micron size is widely used in multi-wire drivers for creation of plasma shells accelerated by electromagnetic forces to the load axis and is basis of pulse sources of x-ray radiation from fast Z pinches [1]. Nanosecond electric explosion of micro-wires in vacuum at slow (20 A/ns) and fast (150 A/ns) energy input, according to experiments [2], has the following features: (1) the sausage-type instabilities are absent; (2) a conductor explosion begins at the moment of switching of electric current to the quickly extending plasma corona; (3) a polarity effect that points to influence of the electron thermal emission from a wire on ionization of environmental vapor [3]; (4) stratification of an extending gas-plasma core is observed after switching the current to the corona. Existing models of electric explosion of conductors and of their stratification do not take into account these features [4, 5, and 6].

Our goal is to construct and investigate the models of plasma corona formation and stratification of the exploding micro-wires, which take into account above-mentioned features of their explosion.


According to the one-dimensional magnetohydrodynamical simulation [7], for a plasma corona formation at explosion of aluminum wire of diameter 25 µm there is enough evaporation of ~30 atom layers (0.1–0.2% of weight of a wire). It is obvious, that in this case applicability of magnetohydrodynamical approach to the vapor dynamics description and its electric breakdown demands careful discussion.

In [3] it is shown, that the radial electric field along a wire axis is not uniform, varying from 0 up 10⁷ V/cm (the peak value of $E_r$ is located near to the anode, and near to the cathode it may even be the negative). Dependences of a thermoelectronic current for tungsten are shown in Fig. 1. They were calculated according to Murphy-Good formula $j = j_{RS} a / \sin(a)$ ($a = 0.5 \pi T_2 T^{-1}$, $T_2 = 1.1 \cdot 10^{-2} E_r^2$, $j_{RS} = 1.2 \cdot 10^{-7} T^2 \times \exp \left( -1.16 \cdot 10^4 \left( \varphi - 3.62 \cdot 10^{-4} \sqrt{E_r} / T \right) \right)$ – the Richardson-Shottky current, $\varphi$ is a work function, $[T] = [T_2] = K$, $[\varphi] = eV$, $[E_r] = V/cm$, $[j] = [j_{RS}] = A/cm^2$), from temperature for different values $E_r$: 1 – $E_r = 0$, 2 – $E_r = 10^4 V/cm$, 3 – $E_r = 10^5 V/cm$, 4 – $E_r = 10^6 V/cm$, 5 – $E_r = 10^7 V/cm$.

From this figure we notice that the electron thermal emission is, practically, absent at temperature less than 2500 K. At melting temperature of tungsten $T_m = 3656 K$ and $E_r = 10^7 V/cm$ the thermoelectronic current is $j = 3.755 \cdot 10^4 A/cm^2$. The energy density of electrons emitted from a wire surface is equal to $j_r = e^{-1} (e_{in} + kT(1 + actg(a)))$ where $e_{in}$ is an energy corresponding with maximum of a distribution function of emitted electrons at $E_r = 0$. As a result of cooling wire surface by emitted electrons in a wire there appears a nonuniform along axis thermal flow directed toward wire surface.

Estimates have shown that magnetic induction in the experiments of authors [2] was ~50 T and the Larmor radius was $r_L = 0.1 \mu m$. It is follows that the emitted electrons are localized near a micro-wire surface forming nonuniform along its axis electron shell. This enables us to investigate of electron dynamics in the framework of electron magnetohydrodynamics [9] generalized to case of varying particle number. It varies as a result of electron thermal emission and ioni-
zation of neutrals evaporated from wire surface, and also of loss of the electrons localized at a distance from wire surface less than $2r_L$. In addition, our estimates point to the fact that it is necessary to take into account a contribution of the electron cyclotron radiation in energy balance of a wire surface.

Simple estimates point to the fact that it can be neglected of the field emission of ions from a wire surface for values of the radial electric field being characteristic for the nanosecond electrical explosion of micro-wires [10]. Thus atoms C and H absorbed on wire surface [11], and also own atoms evaporated from it are matter of the plasma corona. A boundary of the phase transition “incompressible liquid – vapor” presents a moving discontinuity for which equalities of temperatures and chemical potentials of a liquid ($T_L$, $\mu_L$) and vapor ($T$, $\mu$), and also the integral conservation laws of mass, momentum and energy are applicable. As for the “one-phase” approximation used by us, which is applicable up to density values $\rho \leq 0.03\rho_0$, it is suggested that specific energy of the phase transition “liquid-vapor” $Q_p$ is constant in a temperature interval $T_0 \leq T < T_c$ ($T_0$, $T_c$ are, respectively, the temperature of an equilibrium phase transition at pressure $T_0$ and critical temperature) [12]. Temperature of the “liquid-vapor” boundary, $T$, its phase velocity, $D$, and normal component of the vapor velocity, $v$, are equal to

$$T = T_0(1-w)/(1-\alpha_w \ln(P_0^{P_0^{-1}})),$$

$$D = -(q_L + I_e + j_r - j_tr - q_p)/(\rho v Q_p),$$

$$v = (q_L + I_e + j_r - j_tr - q_p)/(\rho Q_p) \, (\alpha_w = RT_p(AQ_p)^{-1}),$$

$$w = p v^2 (\rho Q_p)^{-1},$$

$q_L$, $q_p$ are normal components of the thermal flow in a wire and plasma corona; $j_r$, $j_tr$, $I_e$ are the energy flows of emitted and returned electrons, and also of electron cyclotron radiation from plasma corona. The normal component of momentum, $p = \rho v$, is equal to $p = (q_L + I_e + j_r - j_tr - q_p)/Q_p$. Above obtained expressions are boundary conditions for differential equations describing a vapor dynamics and an electrical breakdown in vapor. We neglect of an electron recombination by electrical breakdown of the vapor shell since processes of shock ionization by emitted electrons are dominant. Also we neglect of contribution of the inelastic collisions in the momentum change. Since vapor extension taking into account ionization and ion elastic collisions with atoms is practically adiabatic. We suppose that temperatures of neutrals and ions are equal. Ion velocities in an electric field and electron velocities in electric and magnetic fields are defined by known expressions [13] connecting them with mobility’s and diffusion coefficients. Limit value of the phase velocity of an ionization wave is the light velocity in vacuum. It is a case of the electromagnetic shock wave. Then we obtain low estimate of the breakdown time (transition time of the electric current in plasma corona), which is $t_b \approx 6.667 \cdot 10^{-11}$ s for wire length 2 cm. In [2, 7] a time of the voltage drop collapse being equal to the transition time of an electric current in plasma corona is $t_b = 8-10$ ns. Then a low estimate of the phase velocity of ionization wave, $D_e = (2-2.5) \cdot 10^8$ cm/s. These estimates point to the fact that in general case it is necessary to take into account finite velocity of heat propagation in energy balance equations for electrons and ions.

According to experiments [2, 7, 11] stratification of a gas-plasma core is observed after transition of a current from a core in a corona. Initial perturbations of a conductor surface are created, in our opinion, at a stage of plasma corona formation. A narrow transition band between a core and a corona is exchanged by a moving discontinuity on which conservation laws are carried out. For simplification of the problem we consider the expansion of a core adiabatic, and its boundary has form as shown in Fig. 2. We suppose, also, that parameters of substance under a bubble and a spike are constant on volume, but not equal to each other. On “bubble–spike” boundary the conservation laws of mass, momentum and energy as on a moving magnetohydrodynamical discontinuity are valid.

![Fig. 2. A form of the core surface](image)

It has allowed us to reduce a problem to system of the ordinary differential equations for coordinate of boundary “core-plasma”, $R$, width, $h_s$, and height, $a_s$, of a spike, and bubble amplitude, $a$. The given system allows investigating nonlinear stages of the Rayleigh-Taylor and Richtmyer-Meshkov instabilities. Transition in a coordinate system moving with velocity of “core-plasma” boundary, gives in occurrence of a pseudo-gravitational field, the bound with acceleration of boundary. It allows us to use for our goals the linear analysis of gravitational instability of the infinite incompressible cylinder, executed in [14]. Using the increment of instability given by the S. Chandrasekhar theory [14], and the experimental estimates of acceleration of “core-plasma” boundary from [2, 11], it is possible to receive estimates of wavelengths most promptly growing perturbations. In Fig. 3 a photo of the core are shown.
The core formed by explosion of an aluminum wire in diameter \( d = 16.2 \, \mu m \) at the moment of time \( t = 93 \, ns \) after a voltage drop maximum and intensity of laser radiation, transiting through a core. The characteristic medial size of strata in experiment has made \( \lambda = 80 \, \mu m \). In Fig. 4 dependences of the characteristic size of strata is given from \( x = kR \) designed under Chandrasekhar theory [14] with use of the experimental estimate of the peak value of acceleration of “core-plasma” boundary. It is visible, that in range of the most growing modes of perturbations \( 0.5 \leq kR \leq 0.65 \) theoretical estimates of characteristic of the strata size are quite well correlated with experimental estimates.

The nonlinear differential equations of the stratification model offered by us not taking into account influences of a plasma corona on dynamics of a gas-plasma core surface were solved by us numerically in range of the wave numbers shown in Fig. 4 and initial form of a surface, shown in Fig. 2. The calculation was carried out for requirements of fast explosion of aluminum wire in diameter 16.2 \( \mu m \) (see Fig. 3). In our calculations the initial temperature was varied from 1 up to 2 eV.

The best fit with experimental value of the velocity of a core boundary \( \sim (4.2–4.3) \times 10^3 \, m/s \) is obtained at \( T \sim (1–1.1) \, eV \). During space-time evolution the initial form of a wire surface changed: the spike and a bubble were interchanged the position. In the process widths of a spike and a bubble remained stationary values (phase velocity of “spike-bubble” boundary was changed in range (0.01–1) \( \, cm/s \)). Fig. 5 gives the initial part of dependence of core-boundary parameters from time: a curve 1 shows \( R_s \) (in initial moment of time it is radius of a spike); 2 – \( R_c \) is a core radius for one-dimensional case; 3 – \( R_b \) (in initial moment of time it is radius of a bubble bottom). It is visible, that as well as in experiment velocity of change of exterior radius promptly enough goes on practically stationary value velocity of expansion of a gas-plasma core, thus radiuses of a spike and a bubble bottom are changed in calculation also with a constant velocity. Owing to the latter in contrast to experiment, radius of a bubble bottom (the curve 1) does not aspire to zero (in experiment as follows from Fig. 3, “bubbles” completely overlaps a core). In our opinion, it points to necessity to take into account an influence of a hot plasma corona pressure and magnetic pressure of the electric current proceeding in a corona on stratification dynamics. In our opinion, it is possible to do within the framework of our simplified piece-continuous description of the two-dimensional hydrodynamics of a flow.

### 3. Conclusion

Thus, we offer models of plasma corona formation (transition of an electric current from a core into a corona) and stratification of a gas-plasma core of exploding micro-wires. The opportunity of use for the description of physical processes in a formed plasma corona of an electronic magnetohydrodynamics is generalized in view of change of particle number as a result of evaporation, ionization and a leaving of electrons on a wire surface. The simplified piece-continuous model for the description of two-
dimensional hydrodynamics of a gas-plasma core of an exploding micro-wire is offered. The carried out computer experiments show that dynamics of exterior radius is well consistent with experiment. Necessity of the account of influence of a hot plasma corona on stratification of a gas-plasma core was grounded.

References