Propagation of an Ionization Shock Wave through Plasma

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Abstract – This paper considers a magnetohydrodynamic model of the plasma with ions of different ionization ratios. Numerical calculations have shown that the propagation of an ionization shock wave through the plasma is accompanied by the generation of directed ion flows at the wave front.

1. Introduction

In high-temperature plasmas, in particular in Z-pinch plasmas, ions of different ionization ratios are normally present. In this case, the forces governed by the thermal and magnetic pressure gradients in the plasma affect the variously charged ions in different ways. This leads to the fact that both ion and electrons acquire diffusion velocities. The effects for which these processes are responsible can be taken into account in the hydrodynamic approximation. For this purpose one should write the Boltzmann equations for each type of ions and electrons and integrate them with different factors [1, 2] that yields the equations presented below.

2. Magnetohydrodynamic Model with Account of Diffusion of Ions of Different Ionization Ratios

Continuity equations. For ions with a spectrometric index $k = 1, ..., k_{\max}$ ($k_{\max}$ = atomic number + 1), the continuity equation has the form

$$\frac{dn_k}{dt} + n_k \nabla \cdot \mathbf{v} + \nabla \cdot \left(\mathbf{V}_k n_k \right) =$$

$$= \frac{n_{k-1}}{\tau^i_{k-1}} - n_k \left(\frac{1}{\tau^r_{k-1}} + \frac{1}{\tau^i_k} \right) + \frac{n_{k+1}}{\tau^r_k}, \qquad (1)$$

where n_k, $\mathbf{V}_k$ are respectively, the number density of k-type ions and their diffusion velocity, i.e., the difference between the macroscopic velocity of ions of this type and the average mass velocities of matter $\mathbf{v}$; τ^i_k, τ^r_k is the ionization and recombination times for k-type ions.

Summation of (1) over ions of all ionization ratios with account of $\sum_k \mathbf{V}_k n_k = 0$ gives an ordinary continuity equation of the form

$$\frac{dn}{dt} + n \nabla \cdot \mathbf{v} = 0 . \qquad (2)$$

Equations of motion. Besides the ordinary equation of motion, which describes the transfer of the total momentum of matter:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla \left(p_i + p_e \right) + \frac{1}{c} \mathbf{J} \times \mathbf{B}, \qquad (3)$$

there appear equations of momentum transfer for ions of each ionization ratio, which can be written in the form:

$$\frac{d\rho_k \mathbf{V}_k}{dt} = -\rho_k \mathbf{V}_k \left(\nabla \cdot \mathbf{v} \right) - \rho_k \left(\mathbf{V}_k \cdot \nabla \right) \mathbf{v} - \nabla p_k -$$

$$-\frac{n_k (k-1)}{n_e} \nabla p_e + \frac{\rho_k}{\rho} \nabla \left(p_i + p_e \right) + \left(\mathbf{R}^i_k + \frac{(k-1) n_k}{n_e} \mathbf{R}^e \right) +$$

$$+\frac{e n_k (k-1)}{c} \left(\mathbf{V}_k - \mathbf{V}_e \right) \times \mathbf{B} + \frac{1}{c} \mathbf{J} \times \mathbf{B} \left(\frac{(k-1) n_k}{n_e} - \frac{\rho_k}{\rho} \right) +$$

$$+\frac{\rho_{k-1} \mathbf{V}_{k-1}}{\tau^i_{k-1}} - \rho_k \mathbf{V}_k \left(\frac{1}{\tau^r_{k-1}} + \frac{1}{\tau^i_k} \right) + \frac{\rho_{k+1} \mathbf{V}_{k+1}}{\tau^r_k}, \qquad (4)$$

where $\rho_k = m_i n_k$; $p_i = \sum_k p_k = \sum_k K T_i n_k$, p_e is the thermal pressure of ions and electrons; $\mathbf{V}_e = \frac{1}{n_e} \sum_k \left(k-1 \right) n_k \mathbf{V}_k$ is the diffusion electron velocity associated with ion diffusion;

$$\mathbf{R}^i_k = \mathbf{R}^{ii}_k + \mathbf{R}^{ie}_k == m_i n_k \sum_l \frac{\mathbf{V}_l - \mathbf{V}_k}{\tau^{ii}_{kl}} +$$

$$+ m_e n_e \frac{\mathbf{V}_e - \mathbf{V}_k}{\tau^{ie}_k} - \frac{m_e}{e \tau^{ie}_k} \mathbf{J}$$

is the friction force acting on k-type ions from the side of ions of the other types and from electrons;

$$\mathbf{R}_e = -\sum_k \mathbf{R}^{ie}_k = n_e m_e \sum_k \frac{\mathbf{V}_k - \mathbf{V}_e}{\tau^{ie}_k} + \frac{m_e}{e \tau^{ie}} \mathbf{J}$$

is the friction force acting on electrons from the side of ions, τ^{ii}_{kl} is the time between collisions of k-type ions with a charge $(k-1)$ and l-type ions with a charge $(l-1)$; τ^{ei}_k is the time between collisions of k-type ions with electrons; $\tau^{ei} = \left(\sum_k \frac{1}{\tau_k^{ei}} \right)^{-1}$ is the full time of electron collisions with ions of all types.

Expressions (3–4) have been written assuming the temperatures of ions of all types to be equal (in a coordinate system related to the average mass velocity of matter) and taking no account of the ion viscosity, anisotropy of the frictions coefficients in a magnetic field, and thermal force.

Expression (4) does not involve spatial derivatives of the diffusion ion velocities and therefore there is no need for solving the equation in partial derivatives to find the diffusion rates $\mathbf{V}_k$. For their determination, we should only solve the system of ordinary linear differential equations for each point of the space.

General Ohm's law. Should we ignore the inertia of electrons in the equation of their motion, a general Ohm law may thus be obtained. In this case, this law has the form

$$\mathbf{E}^* = \frac{\mathbf{J}}{\sigma} - \frac{\nabla p_e}{en_e} + \frac{1}{ecn_e}\mathbf{J}\times\mathbf{B} - -\frac{1}{c}\mathbf{V}_e\times\mathbf{B} + \frac{m_e}{e}\sum_k \frac{\mathbf{V}_k - \mathbf{V}_e}{\tau_k^{ie}}, \quad (5)$$

where $\sigma = \frac{e^2 n_e \tau^{ie}}{m_e}$ is the plasma conductivity; $\mathbf{E}^* = \mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}$ is the electric field in a coordinate system moving with the average mass velocity.

Energy equations. Assuming the temperature of ions of all types to be equal (in the coordinate system related to the average mass velocity of matter), the equations for the internal ion energy can be reduced to the form

$$\rho\frac{d\varepsilon_i}{dt} = -p_i\cdot\nabla\mathbf{v} - \nabla\left(-\kappa_i\nabla T_i\right) + +\sum_k \mathbf{R}_k^i\cdot\mathbf{V}_k + en_e\mathbf{E}^*\cdot\mathbf{V}_e + Q_\Delta, \quad (6)$$

where $\varepsilon_i = \left(\frac{3}{2}KT_i\right)/m_i$ – is the internal ion energy; κ_i is the thermal conductivity coefficient of ions; $Q_\Delta = \frac{3m_e}{m_i}\frac{n_e}{\tau^{ie}}\left(T_e - T_i\right)$ is the heat acquired by ions on collision with electrons due to the temperature difference.

The equation for the internal electron energy has the form

$$\rho\frac{d\varepsilon_e}{dt} = -p_e\nabla\mathbf{U}_e + Q_F - Q_\Delta - -\nabla\left(-\kappa_e\nabla T_e + \rho\varepsilon_e\left(\mathbf{u}+\mathbf{V}_e\right) + \mathbf{W}_r\right), \quad (7)$$

where $\varepsilon_e = \left(\frac{3}{2}T_e + \frac{1}{n_e}\sum_k n_k \sum_l^{k-1} I^l\right)K/m_i$ is the internal electron energy; I^l is the ionization potential of l-type ions; $\mathbf{U}_e = \mathbf{v} + \mathbf{u} + \mathbf{V}_e$ is the total electron velocity; $\mathbf{u} = -\frac{\mathbf{J}}{en_e}$ is the current-induced drift velocity of electrons; κ_e is the thermal conductivity coefficient of electrons;

$$Q_F = -\left(\mathbf{R}^e\mathbf{U}_e + \sum_k \mathbf{R}_k^{ie}(\mathbf{V}_k + \mathbf{v})\right) = = \sum_k \frac{m_e}{e^2 n_e \tau_k^{ie}}\left(\mathbf{J} + en_e\left(\mathbf{V}_k - \mathbf{V}_e\right)\right)^2$$

is the heat evolved in the electron component in electron-ion friction; $\mathbf{W}_r$ is the radiation flux.

Adding expression (6) to (7) yields the expression for the total internal energy:

$$\rho\frac{d\varepsilon}{dt} = -p\nabla\mathbf{v} - p_e\nabla\mathbf{u} + Q_J - -\nabla\left(-\kappa_i\nabla T_i - \kappa_e\nabla T_e + \mathbf{W}_r + (\rho\varepsilon_e + p_e)\mathbf{V}_e + \rho\varepsilon_e\mathbf{u}\right), \quad (8)$$

where $Q_J = \frac{1}{c}\mathbf{V}_e\cdot(\mathbf{J}\times\mathbf{B}) - \mathbf{R}^e\cdot\mathbf{u}$ is the Joule heat. In this case, the Joule heat is evolved not only in electrons, but in ions as well. It includes both the work of friction forces and that of the Hall field governed by ion diffusion.

3. Ion Diffusion During the Propagation of an Ionization Shock Wave through the Plasma

The problem to be solved numerically was that where a piston compresses the Ar plasma along the x-axis. In the plasma, there was a magnetic filed directed along the z-axis. An ionization shock wave is initiated ahead of the piston. The parameters of the plasma where the shock wave propagates were chosen close to the parameters of the Z-pinch plasma and the piston velocity was taken close to the rate of implosion. The initial parameters of the plasma were: $n_{i0} = 10^{17}$ cm^{-3}; $T_{i0} = T_{e0} = 1.5$ eV and piston velocity was $1.5\cdot10^7$ cm/s. The initial ion charge distribution was chosen stationary for the given temperature and density. In so doing, the plasma mainly consisted of neutral atoms and the average ion charge was $\langle Z\rangle \approx 0.7$. Two variants were considered: 1) the thermal pressure ahead of the shock wave front is equal to the magnetic field pressure $\beta_0 = 1$ at an initial magnetic field $B_0 = 3$ kG and, correspondingly, the sound velocity of ions (c_{s0}) approximates the alfven velocity (c_{A0}); 2) the magnetic field pressure ahead of the wave front is considerably higher than the thermal pressure $\beta_0 = 10^{-3}$, $B_0 = 100$ kG and the alfven velocity is comparable with the piston velocity ($c_{A0} \sim 1.1 \cdot 10^7$ cm/s) and more than 30 times higher than the sound velocity of ions. In the first case, the thermal pressure behind the shock wave front was much higher than the magnetic field pressure $\beta_1 \gg 1$ and in the second case $\beta_1 \sim 1$.

Variant 1. The thermodynamic parameters of the shock wave front are presented in Fig. 1. To this case there corresponds a strong shock wave with a Mach number $M \sim 40$ (the ratio of the shock wave front velocity to the sound velocity ahead of the front, $D / \sqrt{c_{s0}^2 + c_{A0}^2}$). It can be seen that high gradients of the thermal pressure, which induce ion diffusion, appear at the shock wave front (the effect of the magnetic field pressure gradient in this case is small).

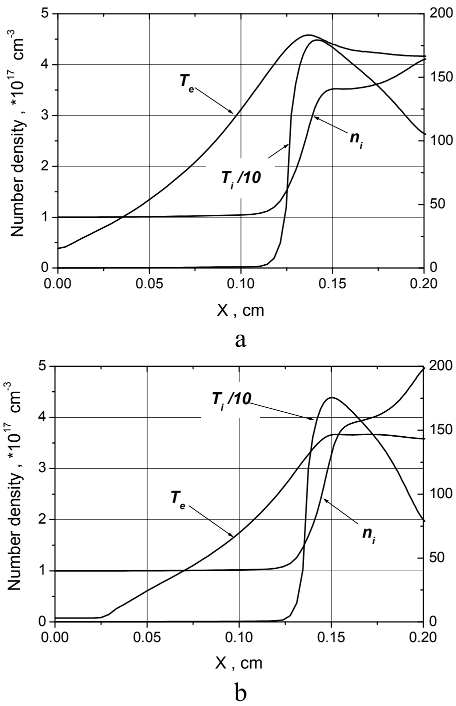

Fig. 1 Spatial distribution of the thermodynamic parameters of the plasma (variant 1): a – with account of ion diffusion; b – without account of ion diffusion

In this case, the rates of ion diffusion have two components. The first component is parallel to the pressure gradient and is perpendicular to the current and to the magnetic field (the x-component). The second one is parallel to the current (the y-component) and is perpendicular to the pressure gradient and to the magnetic field. In the case under consideration, the absolute values of the x-component are more than an order of magnitude higher than the values of the y-component. However, in plasma with an ionization ratio much smaller than unity the situation is the reverse the values of the y-components of the diffusion velocities may exceed those of the x-components.

The direction in which ions diffuse is dictated by their charge. The ions with a charge higher than the average one diffuse in the direction of propagation of the shock wave and the ions with a lower charge diffuse in the opposite direction that is demonstrated in Fig. 2. In so doing, the highest absolute values of the diffusion velocities are displayed by ions of those types whose density is small, as compared to the total density of heavy particles. The spatial distribution of the diffusion rates (the x-component) is shown in Fig. 2,b. As is seen in this figure, the diffusion rates may approach the thermal ion velocity and even be higher than it (see Fig. 4).

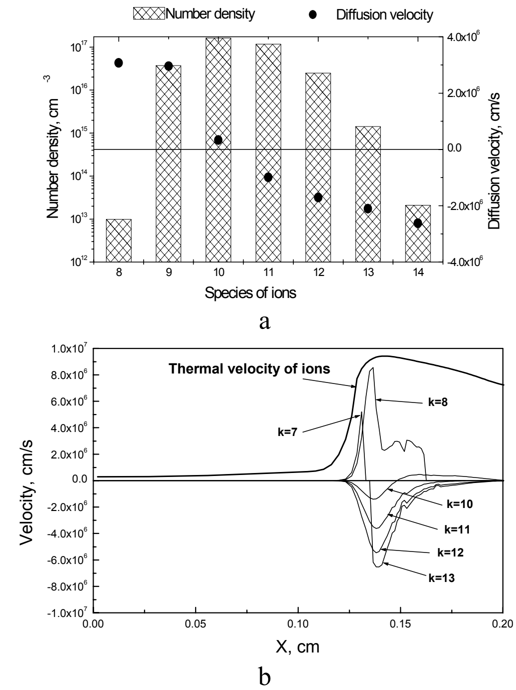

Fig. 2. Distribution of the diffusion velocities (the x-component, variant 1): a – ion type distribution at the point $x = 0.15$ cm (columns – density of ions of a given type; dots – diffusion rate); b – spatial distribution, k – spectroscopic index of an ion

Thus, we can see that the propagation of the ionization shock wave is accompanied by generation of directed ion flows at the wave front. The total velocity of these ion flows includes the average mass velocities of matter, which approximate the piston velocity behind the shock wave front and the ion diffusion rate. It can be seen from Fig. 2,a that the total velocities of the ion flows in the direction of the shock wave propagation are near coincident with the velocity of the shock wave front D which in this case $D \approx 2.1 \cdot 10^7$ cm/s.

The presence of such directed ion flows results in a considerable redistribution of the energy between the ion and electron components beyond the wave front. Without accounting for ion diffusion at the shock wave front the energy is mainly evolved in the ion component due to viscosity. In the electron component, the energy is liberated due to joule heating only in the presence of a magnetic field. With account of ion diffusion, additional heating of the electron component occurs even in the absence of a magnetic field and it is equal to

$$Q_e^f = m_e n_e \sum_k \frac{\left(\mathbf{V}_k - \mathbf{V}_e\right)^2}{\tau_k^{ie}} .$$

In the case in question this value is more than two orders of magnitude higher than the value of the joule energy deposition.

Thus additional heating of the electron component induced by ion diffusion takes place in the ionization shock wave, even in the absence of a magnetic field. This additional heating results in the fact that the electron temperature beyond the wave front turns out to be higher than that without accounting for ion diffusion. Moreover, the additional energy dissipation governed by diffusion leads to expansion of the wave front.

Variant 2. In this case, the thermodynamic parameters of the shock wave are presented in Fig. 3. Here, the propagation of small perturbations ahead of the wave front is determined by the alfven velocity which is comparable with the piston velocity and therefore a rather weak shock wave with a Mach number $M \sim 2.5$ propagates in the plasma. Consequently, the degree of compression of matter beyond the wave front and its temperature are lower than in variant 1.

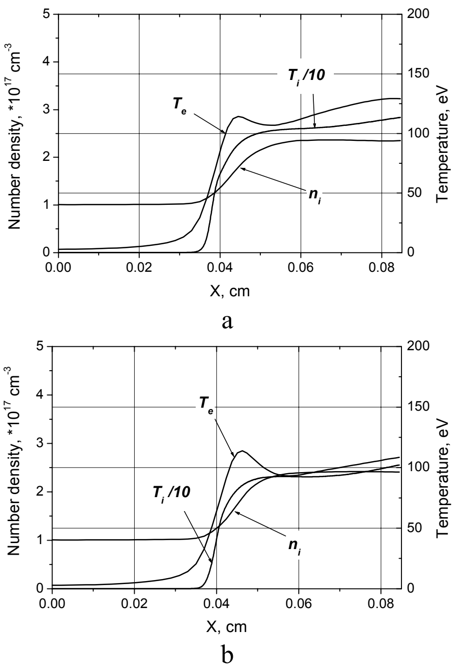

Fig. 3 Spatial distribution of the thermodynamic parameters of the plasma (variant 2): with (a) and without (b) account of ion diffusion

The peak of the electron temperature at the shock wave front is governed by joule heating and by the nonstationary ion content of the plasma. The current induced by the pinched magnetic flow, which heats the electrons, flows in the region where there is an abrupt increase in density and the rate of change of the ion content of the plasma here is lower than the rate of increase in temperature. The drop of the ion temperature in the region where the density changes abruptly is governed by subsequent ionization of the plasma.

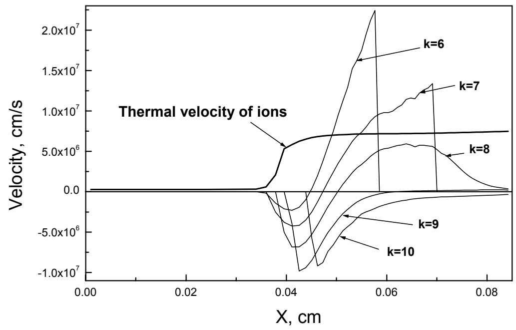

Fig. 4 Spatial distribution of the diffusion velocities (variant 2): k – spectroscopic index of an ion

Because the role of the magnetic field in this case is significant, electron heating is ensured in the main by joule energy deposition and account of ion diffusion leads to smaller changes in the plasma parameters behind the wave front than in variant 1. However, the absolute values of the diffusion velocities in this case are higher and even exceed the values of the thermal velocities of the ion behind the wave front (Fig. 4). As in variant 1, the velocity of ion flows in the direction of propagation of the shock wave (ions with a charge higher than the average one) approximates the velocity of the shock wave front which in this case $D \approx 2.6 \cdot 10^7$ cm/s. Moreover, due to magnetization of the electrons in the wave front the Hall component of the electric field E_x is much higher than the component E_y, Therefore, the work of the Hall field is comparable with the work of friction forces (see (8)).

4. Conclusion

Numerical calculations have shown that the propagation of the ionization shock wave through the plasma is accompanied by generation of directed ion flows at its front. The ion flows with a charge higher than the average one take the same direction as the shock wave and the ion flows with a lower charge take the opposite direction. The velocities of the ion flows in the direction of propagation of the shock wave approximate the wave front velocity.

References

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