Plasma Focus Formation under Short-Running Current Enhancement in Vacuum Discharge[1]

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Abstract – Theoretical consideration is performed of plasma parameters variation along cathode plasma flow of vacuum discharge when a short-term enhancement ("spike") is superimposed on the discharge current. If the current rise rate is high enough then magnetic forces of the current cause a fast compression of the flow towards its axis. It is shown that there is a maximum of plasma acceleration towards axis some distance from the cathode, i.e. the flow shape differs from the initial conical one and a "neck" there is formed. This phenomenon is similar to formation of a "plasma focus" in the convenient high current Z-pinches. It is shown also that a significant plasma heating occurs at the neck under high current rise rate that leads to production of the multiply charged ions even with the discharge current being rather moderate in amplitude (about 1 kA).

1. Introduction

It is well known that action of self magnetic field of pulse discharge current leads to a local constriction of plasma column that results to production there a neck ("plasma focus"). The phenomenon observable just under high values of the discharge current that exceeds approximately 100 kA and leads to production of highly charged ions of the cathode matter in plasma column of vacuum sparks [1, 2]. Nevertheless, the authors of the given paper have observed recently beams of the multiply charged metallic ions (up to Cu^{+19}, for instance) at low voltage vacuum spark with high discharge current rise rate (up to 10^{11} A/s) under the current just being of a few kA [3]. Also, an enhancement in the mean charge state of ions of cathode plasma jet in vacuum arc was observed under application of a short-run spike on discharge current (at range of 100–300 A) of a vacuum arc [4, 5]. The phenomena points to formation at moderate discharge current in cathode plasma jet of a region of hot plasma, where an additional ionization of the cathode matter occurs. In this paper we report studies of conditions, which provide formation of the region. The studies are based on theoretical model that has been developed by the authors earlier [6, 7] and demonstrated a formation of a plasma focus at cathode plasma jet of a vacuum spark.

2. Basic Equations

The hydrodynamic equations for a plasma jet ignoring the viscosity and thermal conductivity are as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0; \qquad (1)$$

$$\frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V}\mathbf{V}) = -\nabla P + \frac{1}{c}\mathbf{j}\times\mathbf{B}; \qquad (2)$$

$$\frac{\partial (\rho\varepsilon)}{\partial t} + \nabla \cdot (\rho\varepsilon \mathbf{V}) = \mathbf{j}\cdot\mathbf{E} - \nabla \cdot (P\mathbf{V} + \tfrac{5}{2}P_e\mathbf{U}) - Q. \qquad (3)$$

Here $\rho = mN_+$, $P = T_eN_e + T_+N_+$ are the plasma density and pressure, respectively; N_e, N_+, T_e, T_+, $\mathbf{V}_e$ and $\mathbf{V}$ are the density, temperature and velocity of ions and electrons $\mathbf{U} = \mathbf{V_e} - \mathbf{V} = -\mathbf{j}/eN_e$ is the current velocity; $P_e = T_eN_e$ is the electron pressure; $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields; $\varepsilon = \frac{3}{2}(T_i + ZT_e)/m + V^2/2$ is the specific ion energy, $Z = N_e/N_+$ is the mean ion charge state; $Q = Q_{\text{ion}}$ is the rate of electron loses that is assumed consisting of ionization loses (Q_{ion}) only.

Set of equation (1)–(3) is added with the Ohm's low

$$\frac{\mathbf{j}}{\sigma} = \mathbf{E} + \frac{1}{c}\mathbf{V}\times\mathbf{B} + \frac{1}{eN_e}\nabla P_e - \frac{1}{ceN_e}\mathbf{j}\times\mathbf{B}, \qquad (4)$$

the charge conservation low

$$\nabla \cdot \mathbf{j} = 0 \qquad (5)$$

and also the Maxwells' equations

$$\nabla \times \mathbf{B} = \frac{4\pi}{\tilde{n}}\mathbf{j}; \qquad (6)$$

$$\nabla \cdot \mathbf{B} = 0. \qquad (7)$$

Here $\sigma = k_\sigma T_e^{3/2}/Z$ is plasma conductivity and c is the light speed.

With equations (4) and (5), the electric field $\mathbf{E}$ is excluded and equation (3) is presented as

$$\frac{\partial (\rho\varepsilon)}{\partial t} + \nabla \cdot (\rho\varepsilon \mathbf{V}) = \frac{j^2}{\sigma} + \frac{1}{c}(\mathbf{j}\times\mathbf{B})\cdot\mathbf{V} - \nabla \cdot (P\mathbf{V} - \frac{5T_e}{2e}\mathbf{j}) - \frac{1}{eN_e}\mathbf{j}\cdot\nabla P_e - Q. \qquad (8)$$

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[1] The work was supported by the Program "University of Russia" under the Grant UR.01.01.008.

In spherical coordinates r, θ, φ with suggestion of axial symmetry of the plasma flow we present the vector parameters as $\mathbf{V} = (V_r, V_\theta, 0)$, $\mathbf{j} = (j_r, j_\theta, 0)$, $\mathbf{B} = (0, 0, B_\varphi)$.

To simplify the problem, the following main assumptions are used.

(i) The current channel and the plasma flow are the same and have the equal cross-section $S = \pi R^2$ where $R(r)$ is their radius varying with a distance z from cathode surface. Thus, we have $\mathbf{j} \times \mathbf{V} = 0$.

(ii) The particle density, pressure and temperature and radial components of ion velocity and current density are taken to be uniform over the flow cross-section. Thus, we assume that these parameters drop to zero in a narrow region near plasma flow boundary.

(iii) Ion pressure is neglected in comparison with electron pressure due to the low value of ion temperature. Thus, $P = P_e$ holds.

(iv)The anode is considered to be a passive collector of the charge and mass fluxes.

(v) The transverse mass and charge flows are negligible, hence the quadratic members of $(V_\theta / V_r)^2$ and $(j_\theta / j_r)^2$.

With integration of eq. (1), r component of eq. (2) and eq. (8) by θ from 0 to the boundary angle $\alpha(r, t)$ one can derive equations as follows:

$$\frac{\partial(\rho S)}{\partial t} + \frac{\partial}{\partial r}(\rho V_r S) = 0; \qquad (9)$$

$$\frac{\partial(\rho V_r S)}{\partial t} + \frac{\partial}{\partial r}(\rho V_r^2 S) = -\frac{\partial P}{\partial r} S; \qquad (10)$$

$$\frac{\partial(\rho \varepsilon S)}{\partial t} + \frac{\partial}{\partial r}(\rho \varepsilon V_r S) =$$

$$= \frac{I^2}{\sigma S} - \frac{\partial}{\partial r}(P V_r S) - \frac{5I}{2e}\frac{\partial T_e}{\partial r} + \frac{I}{eN_e}\frac{\partial P_e}{\partial r} - Q_{\text{ion}} S, \qquad (11)$$

where the flow cross-section is defined as

$$S(r, t) = 2\pi r^2 (1 - \cos\alpha) \approx \pi r^2 \alpha^2. \qquad (12)$$

From eqs. (1), (9) in case of $\rho = \rho(r)$ one can obtain

$$V_\theta(r, \theta) = V_\alpha(r)\frac{\theta}{\alpha}, \qquad (13)$$

where

$$V_\alpha = r\frac{d\alpha}{dt} = r\left(\frac{\partial \alpha}{\partial t} + V_r \frac{\partial \alpha}{\partial r}\right) \qquad (14)$$

is the velocity V_θ at side boundary surface of the flow (i.e. at $\theta = \alpha$). From eqs. (5)–(7) the similar relation for the current density $j_\theta = j_\alpha \theta/\alpha$ and for magnetic field $B_\varphi = B_\alpha \theta/\alpha$ are derived, therewith the values at the boundary surface are as follows: $j_\alpha = j_r r\partial\alpha/\partial r$ and $B_\alpha = 2I/cr\alpha$.

With integration of eq. (2) by θ from 0 to α one derives:

$$\frac{\partial(\rho V_\alpha S)}{\partial t} + \frac{\partial}{\partial r}(\rho V_\alpha V_r S) =$$

$$= \frac{4}{r\alpha}\left(P\,S - \frac{I^2}{2c^2}\right) - \frac{\rho V_\alpha S}{r\alpha}(\alpha V_r + V_\alpha). \qquad (15)$$

From eq. (15) with $V_\alpha = 0$ (a flow with the immobilised side boundary) one derives the well known equilibrium Bennets' condition $PS = I^2/2c^2$. From eqs. (14), (15), the dependence $\alpha(r, t)$ is derived and with eq. (12) the cross-section $S(r, t)$ is also derived. Relations (9)–(11), (14), (15) present the self-consistent set of equations that allows to derive the temporal and space dependencies of the plasma parameters in the cathode flow.

This set of equations should be completed with the kinetic equations for different ion species

$$\frac{\partial f_n}{\partial t} + V\frac{\partial f_n}{\partial r} = k_n f_{n-1} N_e - k_{n+1} f_n N_e, \qquad (16)$$

where $f_n = N_n / N_+$ is an abundance of ion specie with the charge state n and density N_n; E_{n+1} è $k_{n+1}(T_e)$ are the energy and ionization coefficient for $(n + 1)$ ionization degree, respectively. Also, $Z = \sum_{n=1} n f_n$ is the mean ion charge state and $Q_{\text{ion}} = N_e N_+ \sum_{n=1} E_{n+1} k_{n+1} f_n$ is the rate of the ionization lose that is suggested as the only type of loses at present case.

It is clear that if the time of current magnification Δt is less than the ion time of flight $\tau_i \approx L/V_0$ of the interelectrode gap L, then the mass flow G also increases just in vicinity of the cathode, while the discharge current I (that is being carried with electrons, essentially) is magnified immediately, in fact, throughout the gap. The initial mass flow is $G_0(t) = \rho_0 V_0 S_0 = \eta_0 m I(t) / Z_0 e$, where $\eta_0 \approx 0.1$ is the rate of ion erosion. So, with the experimental values of S_0, V_0, η_0, Z_0 (the last one is the mean ion charge state in microjets) and $\alpha_0 \approx \pi/6$, one can derive the complete set of boundary conditions for eqs. (1)–(5).

In the present paper the fast discharge current enhancement was stated with a relation as follows: $I(t) = I_0 + I_a \sin(t/\tau)$, where the current amplitude $I_a = 1$ kA and period $\tau \approx 1$ μs ($\Delta t \approx \tau/4 < \tau_i$) were established closed to the experimental values both in works [3] and in [4, 5]. The initial (at $t = 0$) steady-state discharge current I_0 was assumed closing the interelectrode gap. So, we simulate experimental conditions of work [3], where a pre-plasma of igniter presents at the interelectrode gap before the main discharge pulse being applied. Also, the relation for current $I(t)$ simulates application of “spike” on the discharge current in accordance of experimental conditions of works [4, 5]. To study a dependence of cathode jet pinching on the relative “spike” value, the initial discharge current I_0 was varied.

3. Calculation Results

The set of equations with the boundary conditions presented above was solved numerically with the particle-in-sell method. Let us consider now characteristics of the solution. Fig. 1,a shows that at the initial moment with the discharge current of 100 A the plasma pressure succeeds the magnetic one throughout the cathode flow and the parameter $\beta = 8\pi P / B_\alpha^2 = = 2c^2 PS / I^2$ succeeds a unity. Hence, an influence of the magnetic field on the plasma parameters is negligible.

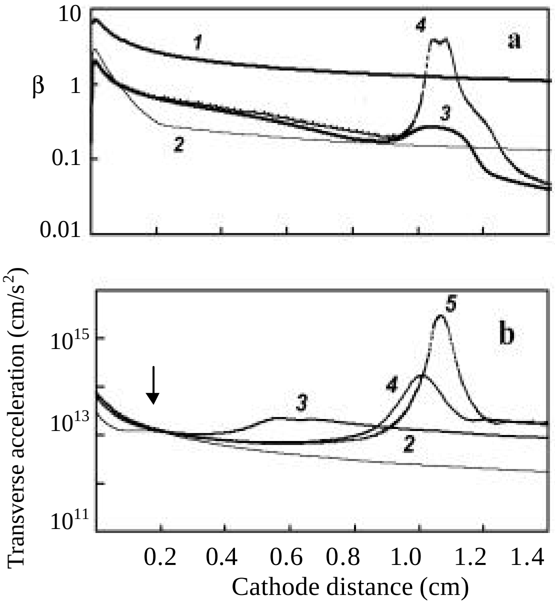

Fig. 1. Ratio between plasma and magnetic pressures $\beta = 8\pi P / B_\alpha^2 = 2c^2 PS / I^2$ (a) and acceleration of side boundary of cathode plasma flow dV_α / dt when its motion towards axis (b) versus cathode distance. Initial and peak discharge currents are 0.1 and 1.0 kA, respectively: *1* – $t = 0$; *2* – 100 ns; *3* – 300 ns; *4* – 490 ns; *5* – 525 ns

The fast current magnification leads to ‘overjumping’ of the equilibrium threshold ($\beta = 1$) and already at 100 ns the magnetic pressure significantly succeeds the plasma one throughout the flow. That causes a compression of the plasma flow with a currents’ magnetic field towards the axis. The different regions of the plasma flow move to the axis with different accelerations $dV_\alpha / dt \approx 1/(\rho\, r)\partial P_B / \partial\theta \approx I^2 / (\rho c^2 R^3)$, where $R \approx r\alpha$ is the cross flow radius. The acceleration near cathode is small because of high density ρ of plasma that is being ejected from the cathode and far apart cathode the acceleration is also small because of low magnetic pressure $\nabla B^2 \propto I^2 / R^3$. Hence there is a maximum of the acceleration at some distance from the cathode. Just at the distance the neck is burn as it is pointed with an arrow in Fig. 1,b. The following development of the neck is determined by competition of two processes: the magnetic compression with the rate time closed to Δt and the drift of the neck with plasma flow that has the characteristic time τ_i. If the current increases rapidly, so that $\Delta t < \tau_i$, then the neck has sufficient time for significant development that leads to a strong plasma compression, its heating and production here the highly charged ions. Fig. 1,b shows also that plasma pressure succeeds the magnetic one (i.e. $\beta > 1$) just at the final stage of neck development and within region of the maximum plasma compression. One can see from Fig. 1 that the stage is in progress rather rapidly, namely during just 20–30 ns the plasma pressure and acceleration rise more that an order of value.

Figure 2 demonstrates variation of side boundary of the cathode plasma flow during the neck development. One can see that rate of the local plasma boundary compression increases with magnification of relative amplitude of current spike even in case of peak of the current being hold. Besides, Fig. 2 shows development of the neck in the process of plasma motion towards anode, so that cross-section of the flow in region of the maximum compression could be less than the initial one.

Formation of plasma focus within a cathode plasma flow leads to a significant electron heating, which, in turn, results in production here of the multiply charged ions of cathode matter. Fig. 3 demonstrates these effects. One can see that at final stage of plasma focus formation the electron temperature in plasma focus attains values exceeding of 100 eV. Also, the relative abundance of ions with $Z = 4$ in this region increases more that an order of value. Fig. 3 shows that length of the stage of intensive plasma heating and production of the multiply charged ions is rather short and it is of the order of ten nanoseconds. Hence, the ions should be observed experimentally just as the short bursts.

6. Discussion and Conclusion

The consideration presented above shows that application of a short-run spike on discharge current results in formation of a neck or plasma focus in cathode plasma flow. This effect is similar, principally, to the well-known formation of plasma focus or “hot point” at plasma column of high-current plasma discharges due to pinch-effect [1, 2]. Nevertheless, there is a set of significant distinctions in the present case. First, plasma focus is formed here under the peak current just at value of a kiloampere. This is near two orders of value less than the corresponding parameter in "classical" *Z*-pinches. The effect is due to, apparently, a presence of a free front boundary of cathode plasma flow that is expanding into the vacuum ambient. The neck is burn just at the region of the boundary, where acceleration of the plasma motion towards the flow axis under the magnetic forces attains its maximum.

Second, it is seen that formation of plasma focus in our case takes place just at sufficiently high discharge current rise rate. It is necessary for the neck to have time for development before the plasma flow carries it off a discharge gap. By the same reason formation of the plasma focus is observable just if the interelectrode gap is of sufficient length. Third, ion drift with the flow velocity through region of hot plasma at focus because of small size of the region leads to a short-term ionization here. This drops the ionization efficiency and results in a non-significant increase in ion charge state even at rather high electron temperature. Nevertheless, the theoretical model considered above is in a qualitative agreement with the experimental data of works [3–5].

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