Theoretical Calculation of Electron Energy Distribution Function in Crossed E×B Fields in Low-Pressure Argon Plasma[1]

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Abstract – In this paper theoretical model and results of analytical calculations of electron energy distribution function (EEFD) for low-pressure argon plasma are presented. In model only elastic collisions that define spectrum of electrons in crossed E×B fields were taken into account. To describe ionization process in plasma we used model approximation. In this approximation the influence of ionization on EEFD in its "tail" was taken into account. The following cases are considered: absence of magnetic field, strong magnetic field and intermediate case. It is shown that at absence of magnetic field the EEFD strongly differs from Maxwellian. In case of a strong magnetic field the electron energy distribution function has distribution close to Maxwellian. Estimations of floating potential of discharge system electrodes in argon plasma in crossed E×B fields with taking into account of non-Maxwellian shape of EEFD are also presented. These calculations were performed to describe low-temperature plasma produced in plasma source based on low-pressure arc discharge with a hollow cold cathode.

1. Introduction

By now the theory of gas discharge is described in detail elsewhere [1–3]. But models described in these books as a rule are presented in common aspect, and they require additional correction for specific experimental applications. Thus the purpose of this work is the construction of model of low-pressure discharge burning in argon. For this reason we used model ionization cross-section close to real ionization cross-section of argon that increases with increasing of average energy of electrons in discharge. Suggested model could be used for description of processes taking place in argon plasma generated by plasma source based on arc discharge with cold hollow cathode immersed in longitudinal magnetic field [4].

2. Main Assumptions and Equations of Model

In developing model we took into account only elastic collisions that define spectrum of electrons in crossed E×B fields. To describe ionization processes in plasma, we used model approximation. In this approximation the influence of ionization on EEFD in its "tail" was taken into account. Approximation of real cross-sections is shown in Fig. 1.

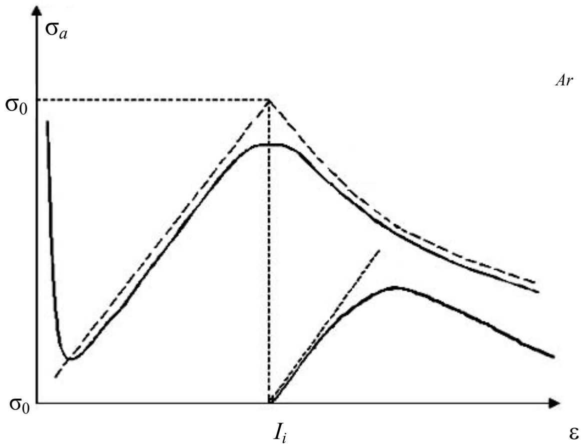

Fig. 1. Electron energy *vs* transport cross-section of electron-atom collisions and ionization cross-section: solid lines – experimental dependence of transport cross-section of electron-atom collisions and ionization cross-section in argon, dashed lines – model approximations

1. The transport cross-section of electron-atom collisions was approximated by expression (1), where σ_0 is maximum transport cross-section for argon ($\sigma_0 = 5.4 \cdot 10^{-19}$ m^2) [2]:

$$\sigma_a(\varepsilon) = \sigma_0 \times \begin{cases} \varepsilon / I_i, \varepsilon \in (0, I_i) \\ I_i / \varepsilon, \varepsilon \in (I_i, \infty) \end{cases}. \quad (1)$$

2. The ionization cross-section near threshold energy was approximated by linear function (2), where $\sigma_{i0} \approx 3 \cdot 10^{-20}$ m^2 for argon [2]:

$$\sigma_i(\varepsilon) = \sigma_{i0} \times (\varepsilon / I_i - 1). \quad (2)$$

3. In this case we have limitation on validity of our results in region $\langle \varepsilon \rangle \langle I_i \rangle$.

In the framework of model the shape of EEFD was calculated and estimation of average electron energy and argon ionization rate was made.

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3. Electrons Energy Distribution Function (EEFD)

We used well-known solution of Boltzmann kinetic equation [5] to investigate influence of magnetic field on EEFD. This solution has the following form:

$$f_0(v) = A\cdot\exp\left\{-\int_0^v \frac{mvdv}{kT+\frac{1}{3}M\left(\frac{eE_\perp}{m}\right)^2\frac{1}{\omega^2+v_a^2}+\frac{1}{3}M\left(\frac{eE_\parallel}{mv_a}\right)^2}\right\}, \quad (3)$$

where kT is the gas temperature, M and m are atom and electron masses, respectively, $\omega = qB/m$ is the electron cyclotron frequency, $\upsilon_a = \sigma_a(\varepsilon)n_a v$ is the transport collision frequency of electrons with atoms, $E_\perp$ and $E_\parallel$ are the transverse and longitudinal (relatively to magnetic force line) projections of electric force vector.

Assuming fields are high enough, we assign $kT \to 0$. We also assume that longitudinal component of electric force is absent. Taking into account these simplifications, the expression (3) can be written as follows:

$$f_0(v) = A\cdot\exp\left\{-\int_0^v \frac{3(\omega^2+v_a^2)mvdv}{M\left(\frac{eE_\perp}{m}\right)^2}\right\} \quad (4)$$

Then, using expression (1) and introducing $\varepsilon = mv^2/2$ and dimensionless definition of energy

$$w = \sqrt[4]{\frac{3m}{2M}\left(\frac{\sigma_0 n_a}{eE_\perp I_i}\right)^2}\,\varepsilon, \quad (4')$$

the expression (4) can be written as follows:

$$F(w) = \frac{\sqrt{w}\exp\left\{-\frac{\gamma^2}{\alpha^{3/2}}w - w^4\right\}}{\int_0^\infty \exp\left\{-\frac{\gamma^2}{\alpha^{3/2}}w - w^4\right\}\sqrt{w}dw}. \quad (5)$$

Constant A in expression (4) we have estimated from requirement of normalization $\int F(w)dw = 1$. Thus, we have expression (5) for main part of EEFD in magnetic field. In this expression there are two dimensionless parameters α and γ, that depend on electric and magnetic field in the following way:

$$\alpha = \sqrt{\frac{2M}{3m}}\frac{eE_\perp}{\sigma_0 n_a I_i} \text{ and } \gamma = \sqrt{\frac{2}{mI_i}}\frac{eB}{\sigma_0 n_a}. \quad (6)$$

Combination $\gamma^2/\alpha^{3/2}$ of these two parameters completely defines the shape of EEFD in argon plasma.

Calculated curves for different values of parameter $\gamma^2/\alpha^{3/2}$ are presented in Fig. 2. It can be shown that increasing of parameter $\gamma^2/\alpha^{3/2}$ leads to increasing of relative fraction of high-energy electrons. But at the same time the average energy of electrons in discharge is decreasing.

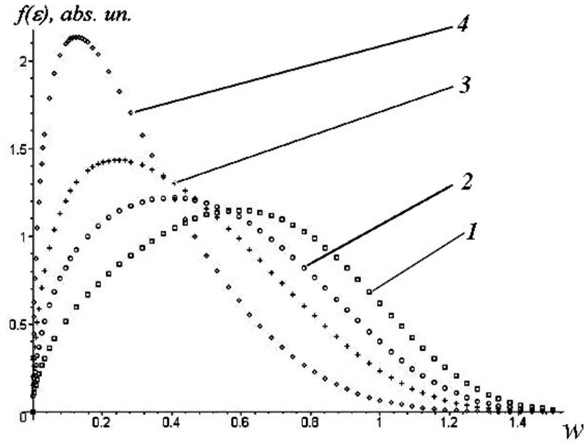

Fig. 2. Shape of EEFD *vs* parameter $\gamma^2/\alpha^{3/2}$ while α = const. Curve *1* corresponds to $\gamma^2/\alpha^{3/2} = 0$, curve *2* – $\gamma^2/\alpha^{3/2} = 1$, curve *3* – $\gamma^2/\alpha^{3/2} = 2$, curve *4* – $\gamma^2/\alpha^{3/2} = 4$

Using (6) and rewriting (5), we can estimate average energy of electrons as follows:

$$\frac{\langle\varepsilon\rangle}{I_i} = \sqrt{\alpha}A\left(\frac{\gamma^2}{\alpha^{3/2}}\right)\times\int_0^\infty w^{3/2}\exp\left\{-\frac{\gamma^2}{\alpha^{3/2}}w - w^4\right\}dw. \quad (7)$$

Results of calculations of (7) are presented in Fig. 3.

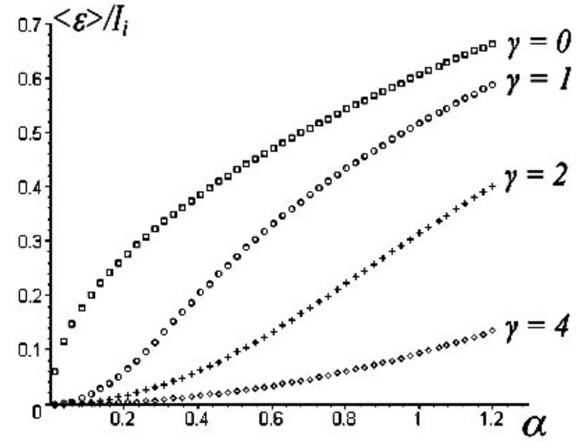

Fig. 3. Average energy *vs* α at different γ

We can estimate the average energy from Fig. 3 if we know values of parameters γ and α.

4. Floating Potential of a Probe in Argon Plasma

Because EEFD is non-Maxwellian the interpretation of probe characteristics in argon plasma is very complicated problem. Here we will go into a problem of calculation of floating potential of the probe immersed in argon plasma with magnetic field. Usually experimentalists estimate value of average energy of electrons using value of a floating potential.

Because in argon EEFD strongly differs from Maxwellian the expression for electron current from plasma on probe will be noticeably differ from Boltzmann expression. Now we introduce dimensionless variable Ψ of a wall potential by analogy with dimensionless energy (7):

$$\Psi = \frac{qV}{I_i\sqrt{\alpha}}\,. \tag{8}$$

It can be shown that value of electron current density on negative charged up to potential (–V) wall can be written by expression

$$j_e(\Psi) = n_0\sqrt{\frac{2I_i}{m}}\alpha^{1/4}A\left(\frac{\gamma^2}{\alpha^{3/2}}\right)\times$$

$$\times\int_{\Psi}^{\infty}(x-\Psi)\exp\left\{-\frac{\gamma^2}{\alpha^{3/2}}x - x^4\right\}dx. \tag{9}$$

We also took into account non-Maxwellian shape of EEFD when we calculated ion current on negative charged wall. In this case it is necessary to define a velocity of directed movement of ions at the entrance to the near-wall region of space charge. In case of Maxwellian EEFD this velocity is named as Bohm's velocity. In non-Maxwellian plasma the ion energy still have the order of average electron energy in plasma. Setting the ion current is equal to electron current we can estimate the value of the floating potential of the wall.

As the result of these calculations the dependence of ratio of floating potential qV_f on average electron energy in argon plasma versus dimensionless parameter $\gamma^2/\alpha^{3/2}$ characterizing the shape of EEFD was obtained. This dependence is presented in Fig. 4.

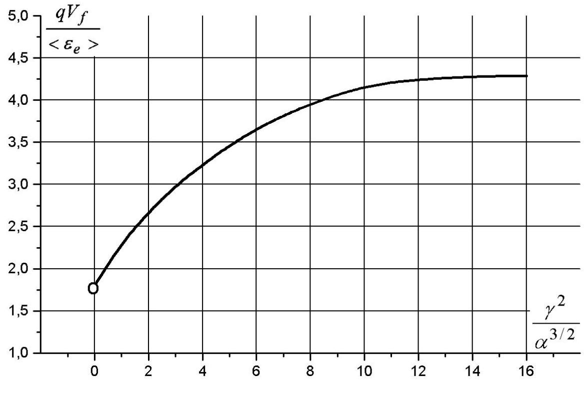

Fig. 4. Dimensionless floating potential *vs* parameter $\gamma^2/\alpha^{3/2}$

For plasma without magnetic field (this point is marked by circle on the figure) we have $qV_f/\langle\varepsilon\rangle \cong$ 1.8. This value is less that one has estimated usually. So for Maxwellian shape of EEFD the ratio $qV_f/\langle\varepsilon\rangle$ in argon would be equal to ≅ 5.3. This is the reason why in argon plasma without magnetic field the floating potential is 1.8 times greater than average energy of electrons. While the shape of EEFD become Maxwellian with increasing of magnetic field the relative magnitude of floating potential also increase relative to average energy of electrons.

5. Estimation of Ionization Rate

To estimate value of ionization rate, it is necessary to make the calculation of EEFD in the region of its "tail" taking into account the ionization process, i.e. the case when $\varepsilon > I_i$. We used method of quasiclassical approximation at integration of Bolzman's kinetic equation [5]. In this case we have following expression for "tail" of EEFD:

$$f(\varepsilon > I_i) \approx F(\varepsilon = I_i)\times$$

$$\times\exp\left\{-\left(\frac{m}{eE}\right)\int_{\sqrt{\frac{2I_i}{m}}}^{v}\sqrt{3\left(\omega^2+\nu_a^2\right)\frac{\nu_i(\varepsilon)}{\nu_a(\varepsilon)}}dv\right\}, \tag{10}$$

where $F(\varepsilon = I_i)$ – main part of EEFD (5), when $\varepsilon = I_i$. Here ν_a is the transport electron-atom collision frequency and ν_i is the electron-atom ionization frequency:

$$\nu_i(\varepsilon) = vn_a\sigma_i(\varepsilon) = vn_a\sigma_{i0}(\varepsilon/I_i - 1),$$
$$\nu_a(\varepsilon) = \sigma_0\frac{I_i}{\varepsilon}n_a v\,. \tag{11}$$

Using (6) and (9) the expression (10) could be written as:

$$f\left(y = \frac{\varepsilon}{I_i} > 1\right) \approx F(y=1)\times$$

$$\exp\left\{-\frac{1}{\alpha}\sqrt{\frac{2M\sigma_{i0}}{m\sigma_0}}\int_1^y\sqrt{\left(\frac{\gamma^2}{4}+\frac{1}{x}\right)(x-1)dx}\right\}. \tag{12}$$

The expression for ionization rate is

$$K_i = \langle\sigma_i v\rangle = \int_{\sqrt{2I_i/m}}^{\infty}\sigma_i(\varepsilon)vf(\varepsilon)4\pi v^2 dv\,. \tag{13}$$

Using (12) and (13) we obtain symbolic expression for estimation of ionization rate:

$$K_i = \sigma_{i0}\sqrt{\frac{2I_i}{m}}A\left(\frac{\gamma^2}{\alpha^{3/2}}\right)\times\alpha^{-3/4}\times\exp\left\{-\frac{\gamma^2+1}{\alpha^2}\right\}\times$$

$$\times\int_1^{\infty}(y-1)y\exp\left\{-\frac{1}{\alpha}\sqrt{\frac{2M\sigma_{i0}}{m\sigma_0}}\int_1^y\sqrt{\left(\frac{\gamma^2}{4}+\frac{1}{x}\right)(x-1)dx}\right\}dy \tag{14}$$

and parameters α and γ are defined earlier in (6).

Calculated curves for the ionization rate versus ratio E/N at constant magnetic field are presented in Fig. 5.

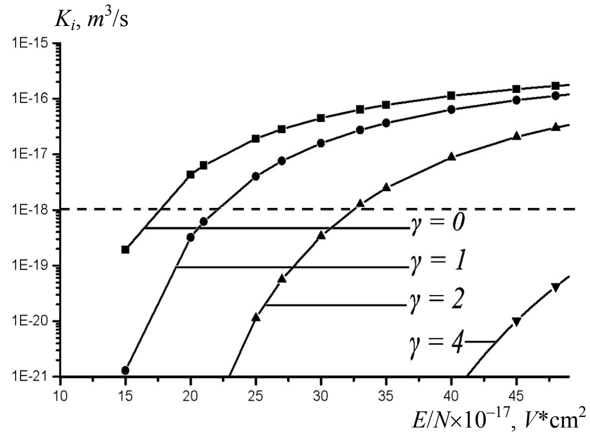

Fig. 5. Calculated ionization rate vs ratio E/N at constant magnetic field. The dashed line is the sample of ionization rate level for state breakdown of argon

It is obvious that increasing of magnetic field leads to increasing of static breakdown voltage for argon to provide ionization rate on the level that is necessary for breakdown.

5. Conclusion

1. Theoretical model for calculation of ionization rate of argon in low-temperature plasma is presented. The model allows estimating average energy of electrons and ionization rate in broad range of parameters (B/n_a) and (E/n_a).

2. It is shown that at the presence of magnetic field the shape of EEFD is strongly deformed; moreover, the part of high-energy electrons increases with increasing of magnetic field.

3. The method of floating potential for the definition of average electron energy needs further comment in case of crossed E×B fields. The ratio between floating potential and average electron energy depends on dimensionless parameter $\gamma^2/\alpha^{3/2}$ that controls the shape of EEFD in argon plasma.

4.The influence of magnetic field on ionization rate is nontrivial: in one hand the increasing of magnetic field at the same electric field leads to decreasing of average energy of electrons but in the other hand this leads to increasing of high-energy electrons part. This is the reason why the static breakdown voltage for argon increases with increasing of magnetic field.

5. In plasma of volume discharge, the increasing of magnetic field leads to increasing of electric field in quasineutral column of discharge.

References

[1] V.E. Golant, A.P. Zhilinskii, I.E. Sakharov, *Basics of Plasma Physics*, Moscow, Atomizdat, 1977, pp. 120–124.

[2] Yu.P. Raizer, *Physics of Gas Discharge*, Moscow, Nauka, 1987, 592 pp.

[3] M.A. Liberman, A.J. Lichtenberg, *Principles of plasma discharges and materials processing,* NY, John Wiley & Sons, Inc., 1994, 572 pp.

[4] L.G. Vintizenko, S.V. Grigoriev, N.N. Koval, V.S. Tolkachev, I.V. Lopatin, P.M. Schanin, Rus. Izv. vuzov. Fizika **9**, 28 (2001).

[5] B.M. Smirnov, *Physics of Low-Ionized Gas*, Moscow, Nauka, 1972, pp. 266–267.