Calculation Procedure of Tangent Stresse's Relaxation Time Using the Results of Dynamic Testing of Metals

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Abstract − The problem has been examined on functional dependency determination of tangent stresses relaxation time on the parameter state of deformable medium. The relaxation time has been introduced by certain interpolation formula containing several free parameter subject to determination on the base of experimental diagrams of dynamic deformation. Experimental data have been interpreted in term of Maxwell model of visco-elastic medium. From mathematical point of view, the procedure of parameter finding for Interpolation formula is optimal control problem. It comes to the minimizing of the functional of mean-square deviation. This functional depends implicitly on relaxation time playing here the role of controlling function. The minimum search of functional of mean-square deviation has been carried out using the method of Nelder-Mead.

1. Introduction

The relaxation time τ is important rheological characteristics of materials. It has been used, when the rheological state equation has been formulated and time-dependent phenomena of irreversible deformation have been explained and mathematically described.

Time relaxation finding is not simple problem. Firstly, time τ is complex and sharply changing function of internal state parameters, loading speed, temperature. The shape of this function is unknown and is determined by extremely pictures of microscopic defects motion. Secondly, there are not experimental methods allowing measuring the relaxation time immediately. Only oblique developments of relaxation phenomena are accessible for measurement and following interpretation. Particularly, it relates to using in this problem macroscopic effect consisting in the distinction of deformation diagrams for various loading velocities and various temperatures. The calculation of deformation diagram is obtained as the result of the numerical integration of one dimensional equations of visco-elastic medium. The relaxation time is included in the coefficients of these equations.

Used here procedure is based on the results of the papers [1–4]. The general approach to the problem under study is described in [1, 2]; the interpolation formula for the relaxation time is obtained on the base of the analysis of the plastic deformation kinetics of metals [5] and presented in that papers. The state equation for series of metals correct for no-ball strain tensor is presented in [3]. Finely, the mathematical model of visco-elastic medium that we use when we treat the experimental diagrams is described in [4].

2. The Mathematical Model and the Treatment Procedure of Experimental Diagram Deformation

The mathematical model of visco-elastic medium of Maxwell's type is described in details in [4]. Besides the conversion law, this model includes the entropy balance equation and evolution equation for the tensor of effective elastic strain ε_{ik}. The latest equation contains the tensor describing the rate of non elastic processes, that is, the relaxation rate. It is necessary to give two scalar functions

$$E = E\left(\varepsilon_{ik}, S\right),\ \tau = \tau\left(\varepsilon_{ik}, S\right) \quad (1)$$

for full identification of the model. The fist of them determines the state equation, the second gives the relaxation time. The state equation allows, using the first thermodynamic law, to calculate the stresses σ_{ik} and temperature T:

$$\sigma_{ij} = \rho\left(\delta_{ik} - 2\varepsilon_{ik}\right)\left(\frac{\partial E}{\partial \varepsilon_{kj}}\right)_S,\ T = \left(\frac{\partial E}{\partial S}\right)_{\varepsilon_{ij}}. \quad (2)$$

The various ways of the state equation parametrization are possible [3, 4]. If the coefficients of stress-strain along the principal axis's k_1, k_2, k_3 are assumed as strain tensor invariants, that is suitable for our problem, we shall obtain

$$\sigma_1 = \frac{\rho_0}{k_2 k_3}\frac{\partial E}{\partial k_1},\ \sigma_2 = \frac{\rho_0}{k_1 k_3}\frac{\partial E}{\partial k_2},\ \sigma_3 = \frac{\rho_0}{k_1 k_2}\frac{\partial E}{\partial k_3}$$

instead the first formula (2). We shall guess further, that the functions (1) are given in the form

$$E = E\left(k_i, S\right),\ \tau = \tau\left(k_i, S\right). \quad (3)$$

The problem on one-axis loading of model specimen (thin rood) underlies the examined procedure. On the assumption of uniform distribution of strains along the rood length (oriented along the axis Ox_1) this problem is reduced to the solution of the equation system

$$\frac{dS}{dt} = \frac{2}{3\tau(k_i,S)T}\left(k_1\frac{\partial E(k_i,S)}{\partial k_1} - k_2\frac{\partial E(k_i,S)}{\partial k_2}\right)\ln\frac{k_1}{k_2},$$

$$\frac{dk_1}{dt} = \dot{\varepsilon}_1 k_1 - \frac{2k_1}{3\tau(k_i,S)}\ln\frac{k_1}{k_2}, \quad (4)$$

$$\left.\frac{\partial E(k_i,S)}{\partial k_2}\right|_{k_2=k_3} = 0\,,\ T = \frac{\partial E(k_i,S)}{\partial S},$$

$$\sigma_1 = \frac{\rho_0}{k_2^2}\frac{\partial E(k_i,S)}{\partial k_1}.$$

That is hybrid equation system including differential equations and final algebraic correlations. The first equation of the system (4) reflects thermodynamical aspects of deformation – entropy change during relaxation process of the tangent stresses. The second equation describes the change of the basic deformation characteristics – effective coefficient of axis tension (pressing) k_1. The third equation expresses the condition of transversal stress absence taking into account the symmetry of the problem $k_2 = k_3$. It is nonlinear algebraic equation from which one can calculate the characteristics of lateral deformation k_2. At last, the latest two equations of the system (4) are implicit or no implicit (depending on the state equation form) algebraic correlations allowing to calculate the temperature T and deformation diagrams $\sigma_1 = \sigma_1(\varepsilon_1)$.

Differential equations are solved at the addition conditions

$$S\big|_{t=0} = 0\,,\ k_1\big|_{t=0} = 1\,,\ \dot{\varepsilon}_1 = \dot{\varepsilon}_1(t) = \text{const}\,,$$

the latest from which means, that hard testing machine is modeled numerically, when constant deformation rate is provided, and the load is recorded magnitude.

Let us assume that theoretical $\sigma = \sigma(\varepsilon\ \tau)$ and experimental $\hat{\sigma} = \hat{\sigma}(\varepsilon)$ deformation diagrams are known (here and further, index "1" near the variables σ_1, ε_1 is omitted for short). Theoretical curve is the solution of the problem (4) and depend on the function τ as on the parameter. Let consider the functional

$$\Phi(\tau) = \frac{1}{\varepsilon_2 - \varepsilon_1}\int_{\varepsilon_1}^{\varepsilon_2}\left[\frac{\sigma(\varepsilon,\tau) - \hat{\sigma}(\varepsilon)}{\hat{\sigma}(\varepsilon)}\right]^2 d\varepsilon\,, \quad (5)$$

meaning geometrically the normalized by some way difference of the areas under deformation curves. This functional is positively defined and bounded below; when the diagrams coincide entirely, it vanishes.

The problem is formulated so: it is need to find the function τ, included in the system (4), when the functional Φ reaches minimum. From mathematical point of view, it is the optimal control problem. The relaxation time τ plays the role of the controlling function. Presenting this function by certain interpolation dependency

$$\tau = \tau(k_i, S, a_1, a_2, ..., a_n)\,, \quad (6)$$

containing n free parameters a_i and substituting the integral included in (5) by some quadrature formula, one can reduce the problem of the functional minimization to the finite dimensional problem of the minimization of the function on n variables

$$F(a_1, a_2, ..., a_n) = \sum_{i=1}^{N} w_i\left[\frac{\sigma(\varepsilon_i, a_1, a_2, ..., a_n) - \hat{\sigma}(\varepsilon_i)}{\hat{\sigma}(\varepsilon_i)}\right]^2.$$

The values of weighting coefficients w_i depend on the type of used quadrature formula.

To find the minimum of the function F we have use the method Nelder-Mead [6], when we have solve numerically the differential equation system (4), we have use the Runge-Kutta's method of four order of accuracy according to recommendations [1].

The dependency

$$\tau = \frac{a_1}{a_2 + a_3\varepsilon^p}\exp\left(\frac{a_4 + \varepsilon^p\left(a_5 + a_6\varepsilon^p\right)}{\sigma_s}\right) \quad (7)$$

had been used as interpolation function (6), where ε^p, σ_s are the intensity of the plastic strains and tangent stresses correspondingly. This formula follows from the analysis of dislocation kinetics of the plastic deformation and is proved in [1, 2]. The parameters a_2, …, a_6 have certain physical sense [5].

3. Numerical Results

We have used the experimental deformation diagrams of aluminum alloy AMg-6 at various temperatures (298–523 K) and deformation rates (less than 2000 c^{-1}) from works [7, 8]. The numerical results (solid and dotted lines) are presented in Figs. 1–8 together with corresponding experimental data (geometrical insignia – square, circle, and triangle). Negative value $\dot{\varepsilon}$ corresponds to tests towards compression, positive relates to tension ones.

Experimental points have been used to determine the optimal parameter set a_i^*, so that $F\left(a_i^*\right) = \min$, i.e. parameter set when numerical and experimental diagrams are maximal close. Typical step of iteration process consists in the following. According to Nelder-Mead's algorithm, there is the polyhedron (simplex) in n-dimensional space, in the apexes of which the function F is known. This polyhedron has been rebuild for each iteration so that the change of the worst apex, where F is maximal by new best one, where F takes the smaller value. As a result, the motion towards the point of minimum a_i^*. Each new point (a_1, a_2, …, a_n) determines, according to (7), new function τ. Using this function the equation system (4) has been solved and new deformation diagram $\sigma = \sigma(\varepsilon, a_i)$ depending on current parameter set a_i has been build.

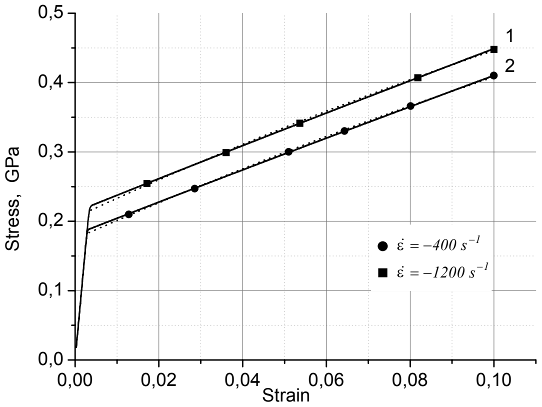

Fig. 1

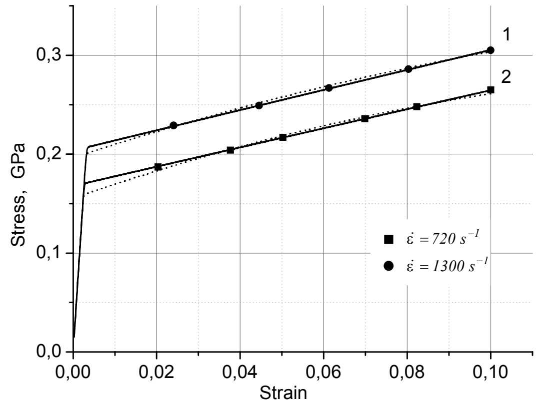

Fig. 2

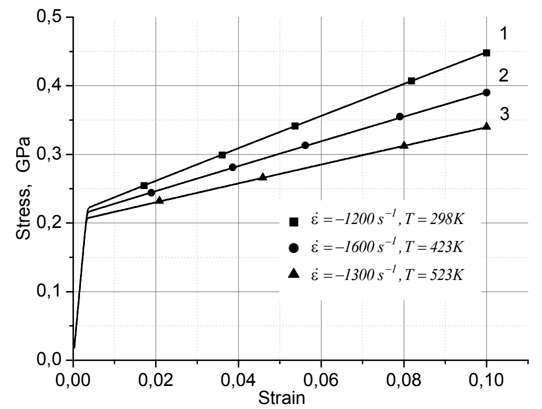

Fig. 3

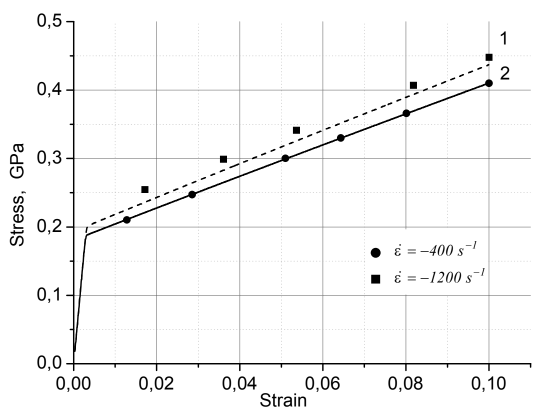

Fig. 4

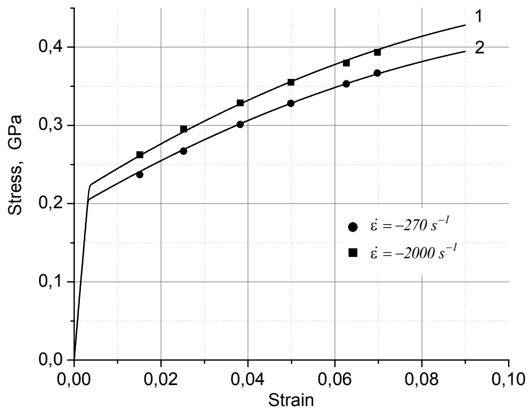

Fig. 5

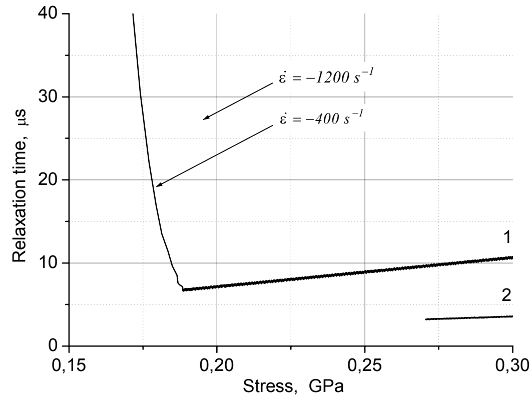

Fig. 6

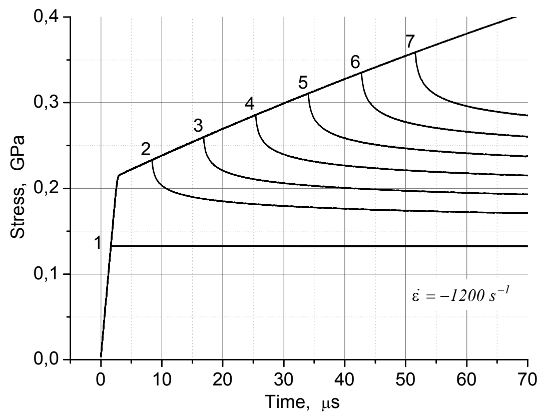

Fig. 7

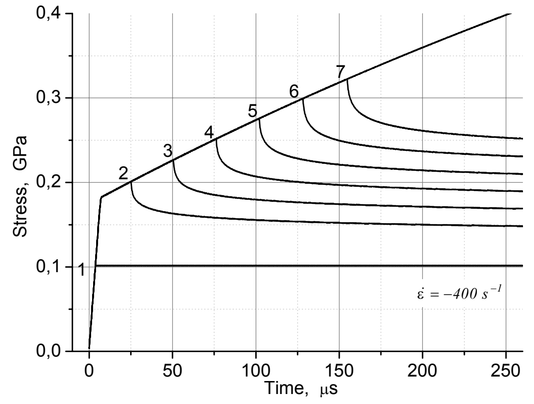

Fig. 8

Then the value of the function of mean-square deviation F has been calculated and the decision, the regular trial is successful or no, has been taken. After that the choice of new point ($a_1, a_2, \ldots, a_n$) is carried out and the process has been repeated. As finely diagram, $\sigma = \sigma(\varepsilon, a_i^*)$ has been assumed, and the parameter set a_i^* corresponding to the minimum point determines the sought dependency (7). The closeness of calculated and experimental diagrams serves as performance criterion.

The deformation curves corresponding to the tension and compression tests are presented in Figs. 1, 2. for the room temperature $T = 298$ K and various loading speeds. The results corresponding to several initial temperatures are shown in Fig. 3 for the closed deformation rates. These results indicate earnestly, that one can approximate arbitrarily exactly each individual diagram $\hat{\sigma} = \hat{\sigma}(\varepsilon)$ for given T and $\dot{\varepsilon}$. It is necessary, however, to note that own parameter set a_i^* different from other curves has been obtained for each experimental curve. If the values a_i^* corresponding to some curve, for example 2, are fixed and used for numerical modeling of test series enclosing the wide diapason of the loading conditions, we shall obtain the qualitatively correct picture. For example, the evaluation of $\dot{\varepsilon}$ gives the curves located higher then experimental curve 2. However, one can expect quantitative agreement of numerical and experimental results only for small vicinity of curve 2, i.e., if $\dot{\varepsilon}$ differs from 400 c^{-1} not very greatly. That is illustrated in Fig. 4, where one can see the dotted curve calculated for $\dot{\varepsilon} = 1200$ c^{-1} with the help of values a_i^* corresponding to the rate $\dot{\varepsilon} = 400$ c^{-1}. This curve deviates essentially from corresponding experimental points.

One can use simultaneously several experimental diagrams, then the functional (5) will include several items, each from which corresponds to individual diagram. In this case, it is succeed to describe satisfactorily both experimental curves shown in Fig. 1 by one set of constants a_i^*, however the approximation accuracy for each separate curve falls visibly. One cannot say that it takes a place in Fig. 3. Defining the minimum point for the functional using three experimental curves, one can see that theoretical curves groups near the curve 2, approximating the experimental points 1 ($T = 298$ K) and 3 ($T = 523$ K) equally badly.

It is clear from above, that entered into interpolation formula (7) coefficients could be guessed as functions of T and $\dot{\varepsilon}$, with the dependency on temperature is more essential then on deformation rate one.

Let us note, that depending on experimental diagram's type, it is expedient sometimes to use no general formula (7), and its simplified variants: $n = 5$, $a_6 = 0$ (linear hardening), and $n = 4$, $a_5 = 0$, $a_6 = 0$ (fluidity area, ideal plasticity). General formula describes well linear hardening at $n = 6$ [8], as it is shown in Fig. 5, but gives worse results for linearly hardening materials (Fig. 1, 2, dotted line), then more simple variant $n = 5$ (Fig. 1, 2, solid line).

Time relaxation change along numerical curves 1, 2, presented in Fig. 1, is shown in Fig. 6. It is shown that the function τ changes more essentially near the vicinity of elastic-plastic transition.

The relaxation curves are presented in Fig. 7, 8. Aluminum rood is loaded to ε of certain value, further the deformation is fixed and sustained for the same level. The curves 2–7 reflect the exponential attenuation of the stresses at the fixed value of strains. The stress relaxation does not occur when the elastic deformation stage is observed (line 1).

4. Conclusion

Optimal values of interpolation coefficients are determined in developed variant of computer procedure by one are several deformation diagrams. In the last case, each experimental diagram can be included in functional with own statistical weight, taking into account the accuracy of experimental data.

In distinct to [1, 2], this method is added by the algorithm of the minimization of the functional of mean-square deviation and automatized fully. This method is universal enough and allows evaluating practically the capacity for work and flexibility of interpolation formula of arbitrary view.

The calculations show that the interpolation formula (7), constructed on the base of dislocation micro mechanism of plastic deformation, allows to approximate with enough accuracy the dynamic deformation diagrams in the comparatively narrow diapason of the loading parameter changes. The approximation of wide-range series of experimental curves could be carried out enough accurately, if the parameters of interpolation formula are taken as the function of the temperature and deformation rate.

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