Three-Dimensional Mathematical Model of Thermal Treatment of Hollow Cylinder with Coating

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Abstract – The mathematical model of the process of thermal treatment of lateral surface of detail in the form of hollow cylinder has been suggested. Effective energy source corresponds to laser irradiation heating or scanning electron beam and moves along spiral trajectory on the cylinder surface with given velocity. The model has been formulated in cylindrical coordinate system. The temperature field, the sizes of molten pool and heat-affected zone, thermal cycles for chosen points on the surface and in depth of detail has been determined during numerical solution for various time moments depending in external source parameters, its velocity, thickness and properties of he base and coating.

1. Introduction

The choice of processing regime during material treatment technologies depends on the form and size of treated detail and on the properties and composition of materials. The idealize concepts on the thermal processes and connected with them processes of properties transformation following from the simplest theoretical models can have no place for the details of real form and sizes or will at least sensible to geometrical factors. Additional difficulties appear in the result interpretation, if the surface is coated from material with the properties differ from base ones. Suggested model can be used for the investigation of thermal treatment dynamics of detail in the cylindrical form and extends the paper [1].

2. Problem Formulation

Let us consider the problem in the following formulation. Let source with the energy distributed by specified law moves along external lateral surface of the cylinder with internal radius R_1 and external radius R_2 (Fig. 1). The heating source moves along the treated surface with some velocity V. If effective source of external energy corresponds to laser irradiation heating, the capacity density in them is distributed by Gauss law. In this case, we assume that the substances are opaque for laser irradiation. If electron beam is used for thermal treatment, the energy distribution in the effective source depends on scanning regime. We take into account that one can neglect the electron penetration in substance depth that is correct for real capacity density used in the technologies. The coating can be deposited previously on the detail, the sickness h and properties of which are considered known.

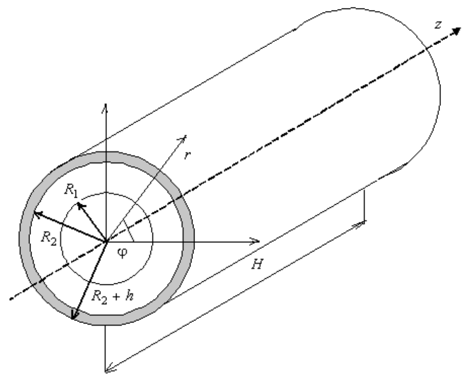


Fig. 1. Illustration to problem formulation

The mathematical formulation of the problem includes the thermal conductivity equation in cylindrical coordinate system

$$c\rho\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{J}\ ,\ \ \mathbf{J} = -\lambda \nabla T\ , \tag{1}$$

where

$$\nabla T = \frac{\partial T}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial T}{\partial \varphi}\mathbf{e}_\varphi + \frac{\partial T}{\partial z}\mathbf{e}_z\ ,$$

$$\nabla \cdot \mathbf{J} \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(rJ_r\right) + \frac{1}{r}\frac{\partial}{\partial \varphi}J_\varphi + \frac{\partial}{\partial z}J_z\ ,$$

J_r, J_φ, J_z are the components of the vector of heat flux density, $\mathbf{e}_r$, $\mathbf{e}_\varphi$, $\mathbf{e}_z$ are unit vectors of cylindrical coordinate system; c, ρ, λ are thermal physical properties (heat capacity, density and thermal conduction coefficient) depending on temperature and coordinate r;

$$r = R_1 : \frac{\partial T}{\partial r} = 0\ ; \tag{2}$$

$$r = R_2 + h\ :\ -\lambda\frac{\partial T}{\partial r} = q\left(\varphi, z, t\right) + \alpha\left(T_e - T\right)\ ; \tag{3}$$

$$z = 0, H : \frac{\partial T}{\partial z} = 0\ ; \tag{4}$$

$$t = 0 : T = T_0\ . \tag{5}$$

Such problem formulation is correct if the thermal contact between the coating and base is ideal and chemical conversions absent.

Assuming that the coating is think in comparison with the base, we integrate the thermal conductivity equation (1) over r in the limits from R_2 to R_2+h taking into consideration the condition (3) and the conditions of ideal heat contact. Than we come to the problem including the equation (1) with the properties of base (c_1, ρ_1, λ_1), the conditions (2), (4), (5) and

$$-\lambda_1 \frac{\partial T}{\partial r} = q(\varphi, z, t) - c_2\rho_2 h \frac{\partial T}{\partial t} - \alpha(T_e - T) -$$

$$-\left[\frac{h}{(R_2+h)R_2} \frac{\partial}{\partial \varphi}\left(\lambda_2 \frac{\partial T}{\partial \varphi}\right) + h\frac{\partial}{\partial z}\left(\lambda_2 \frac{\partial T}{\partial z}\right)\right], \quad (6)$$

where c_2, ρ_2, λ_2 are the coating properties.

The calculations

$$\int_{R_2}^{R_2+h} \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_2 r \frac{\partial T}{\partial r}\right) dr =$$

$$= \int_{R_2}^{R_2+h} \left[\frac{\lambda_2}{r}\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(\lambda_2 \frac{\partial T}{\partial r}\right)\right] dr =$$

$$= \lambda_2 \frac{\partial T}{\partial r}\bigg|_{R_2}^{R_2+h} + \lambda_2 \frac{T}{r}\bigg|_{R_2}^{R_2+h} + \int_{R_2}^{R_2+h} T\frac{\partial}{\partial r}\left(\frac{\lambda_2}{r}\right) dr \approx$$

$$\approx -q(\varphi, z, t) - \lambda_1 \frac{\partial T}{\partial r} - \alpha(T_e - T);$$

$$T(R_2, \varphi, z, t) \approx T(R_2 + h, \varphi, z, t);$$

$$\int_{R_2}^{R_2+h} \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda_2 \frac{\partial T}{\partial \varphi}\right) dr \approx \frac{h}{(R_2+h)R_2}\frac{\partial}{\partial \varphi}\left(\lambda_2 \frac{\partial T}{\partial \varphi}\right);$$

$$\int_{R_2}^{R_2+h} \frac{\partial}{\partial z}\left(\lambda_2 \frac{\partial T}{\partial z}\right) dr \approx h\frac{\partial}{\partial z}\left(\lambda_2 \frac{\partial T}{\partial z}\right)$$

were taken into consideration.

Correspondingly to theoretical conceptions, the heat capacities of the substances rise sharply near by the vicinities of phase transition temperatures. The formulae

$$(c_1\rho_1)_1 = (c_2\rho_2)_{\text{eff}} + L_{\text{ph},1}\rho_{s,1}\delta(T - T_{\text{ph},1}),$$

$$(c_2\rho_2) = (c_2\rho_2)_{\text{eff}} + L_{\text{ph},2}\rho_{s,2}\delta(T - T_{\text{ph},2}), \quad (7)$$

where

$$(c_1\rho_1)_{\text{eff}} = \begin{cases} c_{s,1}\rho_{s,1}, \ T < T_{\text{ph},1}; \\ c_{L,1}\rho_{L,1}, \ T \geq T_{\text{ph},1}, \end{cases}$$

$$(c_2\rho_2)_{\text{eff}} = \begin{cases} c_{s,2}\rho_{s,2}, \ T < T_{\text{ph},2}; \\ c_{L,2}\rho_{L,2}, \ T \geq T_{\text{ph},2} \end{cases}$$

reflect this fact. Here δ is delta-function; $L_{\text{ph},1}$, $L_{\text{ph},2}$ are melting (crystallization) heats; $T_{\text{ph},1}$, $T_{\text{ph},2}$ are melting (crystallization) temperatures for the base and coating; index "L" relates to liquid phase; "s" corresponds to solid phase. (In the real calculations, delta – functions is interchanged by delta-like function satisfying to the normalization requirement). Experimental data on thermal physical properties far from melting temperatures are approximated by polynomial.

3. Numerical Results

The low-carbon steel was used as the base when the calculations were carried out Iron, molybdenum, tungsten, titanium, nickel, copper were used as material for coatings during numerical investigations. Their properties are described in literature very well. Depending on correlation between the properties of materials and their phase transition temperatures various regimes of thermal treatment are observed.

As examples, the results of the calculation of the thermal treatment process of lateral surface of thin ($R_2 = 3$, $R_1 = 2.4$, $H = 20$, $h = 0$) and thick ($R_2 = 10$, $R_1 = 5$, $H = 5$, $h = 0$) cylinders are presented and illustrate the possibilities of the model. The scanning regime is such that the effective source of constant capacity density q_0 occupies on the surface the area 1×1 cm^2. The source returns firstly along front edge of cylinder, and then moves along spiral trajectory with distance between convolutions equal to the source width. In the first case, the source returns during 30 s, so its linear velocity is equal to 0.628 cm/s; in the second case, one rotation period amounts to 50 s, and linear motion velocity is 1.26 cm/s.

In the case of thin cylinder and $q_0 = 5\cdot10^4$ Vt/cm^2 the maximal temperature of lateral surface does not reach the melting temperature. It does not exceed even 850–870 K on the distance 0.3 cm on cylinder surface. Nevertheless, Fig. 2 illustrates well the complex thermal cycles typical for the thermal treatment.

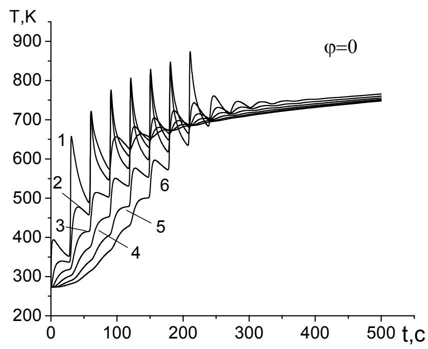


Fig. 2. Dependencies of temperature on time for six points in the section $r = 2.7$ cm; 1 – $z = 1$; 2 – $z = 2$; 3 – $z = 3$; 4 – $z = 4$; 5 – $z = 5$; 6 – $z = 6$ cm

Two dimensional temperature field is presented in Fig. 3 for thin cylinder for various time moments. The pictures are presented through 60 s (i.e. through 2 rotation period), starting with 15 s. The coordinates $z \in [0,20]$ cm and $\varphi \in [0,2\pi]$ corresponds to axises. The latest picture in the right column corresponds to section $r = 2.4$, everything else – to $r = 3$ cm, that is on the surface.

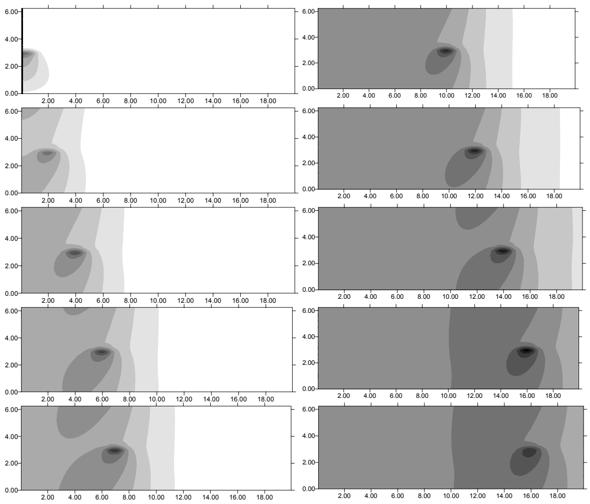


Fig. 3. Temperature field on the surface of thin cylinder during thermal treatment. Black color corresponds to maximal temperature (here, below 1200–1300 K); white color – to temperature below 300 K

Increasing the capacity density of heat source to $q_0 = 6{\cdot}10^4$ Vt/cm^2, the molten pool forms on the cylinder surface (black color in Fig. 4), the form of which changes in time and differs from one, when plane surface of plate or disk is treated. Analogously to previously, the latest picture in the right column corresponds to section $r = 2.4$, everything else – to $r = 3$ cm.

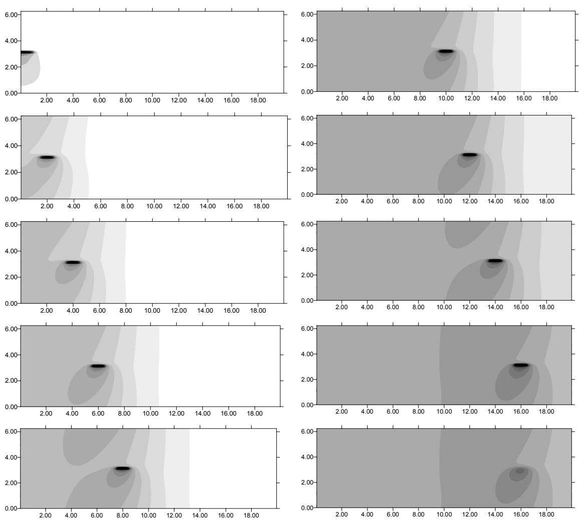
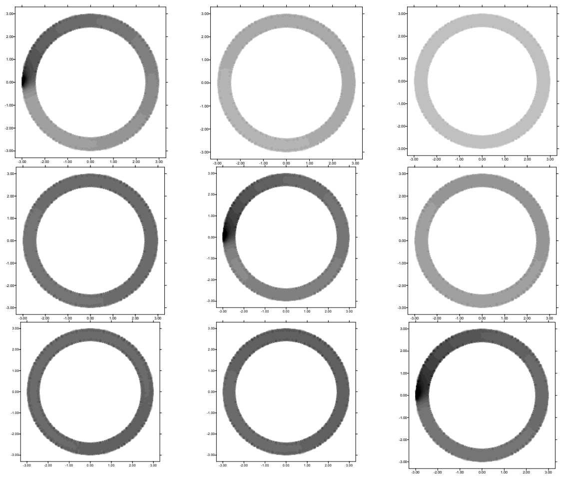


Fig. 4. The temperature field of thin cylinder during thermal treatment (all is analogously to Fig. 3). $q_0 = 6{\cdot}10^4$ Vt/cm^2

Evolution of the temperature field in time in the various sections of the cylinder parallel to its bottom is shown in Fig. 5. With source motion upwards along the axis *Oz*, the cylinder walls are heated more uniform, but maximal temperature diminishes. Black color corresponds here to the melting temperature.



$z = 1$ см $z = 3$ см $z = 5$ см

Fig. 5. The temperature smoothing along thin cylinder thickness; the pictures are presented for the (top-down): 1 – $t = 45$; 2 – $t = 105$; 3 – $t = 165$ s

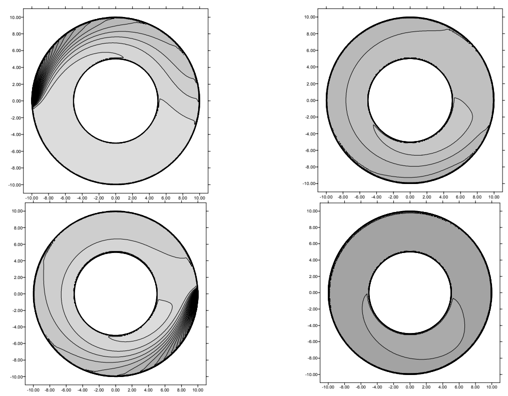


Fig. 6. The temperature smoothing along thick cylinder thickness in the section $z = 1$ cm; the pictures correspond to time moments 1 – $t = 25$; 2 – $t = 50$; 3 – $t = 150$; 4 – $t = 300$ s

The heating-up of thick cylinder is more non-uniform during the motion of the same source along its surface. The isoline concentration in Fig. 6 relates to the region of maximal temperature gradient

Evolution of field temperature during the treatment process is presented in Fig. 7 for the section $\varphi = 0$ through one rotation period for times $t = 25$, 75, 125, 175, 225, 275 s. The radial coordinate corresponds to the abscissa-axis, coordinate z relates to ordinate-axis. Black color corresponds to temperature above 1200 K. Fig. 8 presents the evolution of molten pool (black color, temperature is higher then 1800 K) and heat-affected zone (grey color; temperature is high then 900 K) on lateral surface of cylinder, that is in the section $r = 10$ cm. The pictures are presented in the coordinates z (along the abscissa-axis) – φ (along the ordinate-axis) for the same times. Even to time $t = 275$ s, internal surface of cylinder warm up poorly, that Fig. 9 illustrates. Black color corresponds here to the temperature above 1200 K. All sections of thick cylinder ($r = \text{const}$ or $z = \text{const}$) turn up in various conditions (Fig. 10): the curves $T(t)$ are different.

The temperature field of thick cylinder during thermal treatment

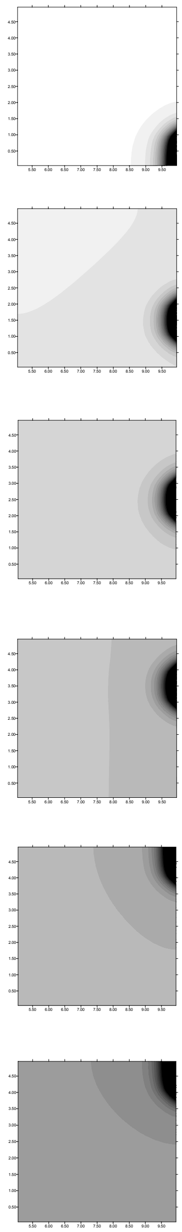


Fig. 7

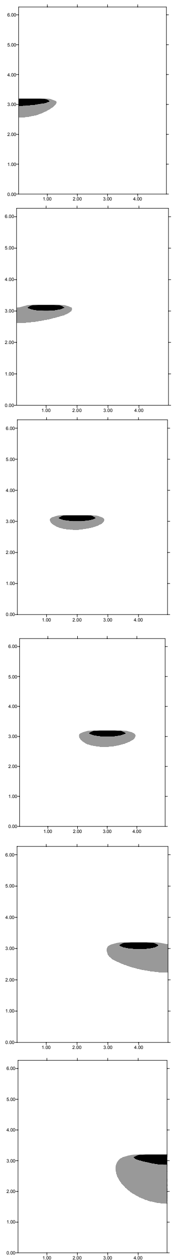


Fig. 8

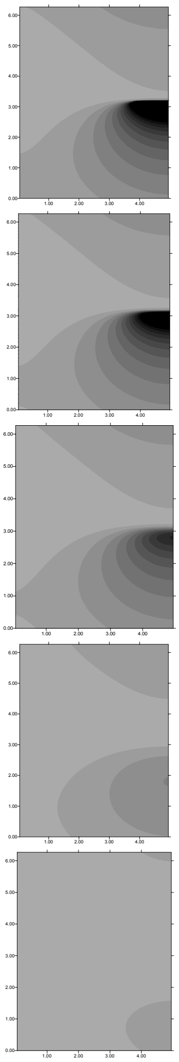
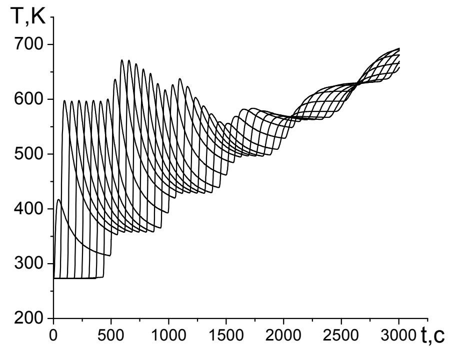
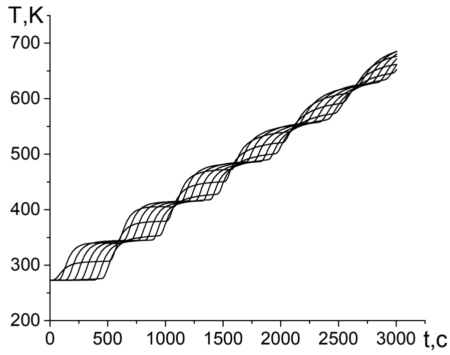


Fig. 9. Sections (top-down): $r = 10$; 9.5; 9.0; 7.5; 5.5 cm, $t = 275$ s



a



b

Fig. 10. Dependencies of temperature on time for some points of cylinder. The curves are presented for angles $\varphi = 0$, 45, 90, 135, 180, 225, 270, 315 (from left to right): a – $r = 9$, $z = 1$ cm; b – $r = 7$, $z = 2$ cm

4.Conclusion

Described above results do not contradict the observed regularities, that can serve for the confirmation of model correctness. The program for numerical solution of the problem can be used for choice of the regime of technology process for known material properties to obtain, for example, the heat-affected zone of required size. The model can be improved taking into account the heat release (heat absorption) during chemical conversions which are possible in the coating that is important for the choice of treatment regimes for the materials with various properties.

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Reference

[1] Knyazeva A.G., Demidov V.N., *in: Proceedings of 12th International conference on radiation physics and chemistry of inorganic materials*, 23–27 September, 2003, Tomsk, TPU, p. 200–204.