The Effect of the Light Beam Diameter of a Laser Pulse on the Critical Energy of Explosive Ignition

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Abstract – An expression for the critical energy density required to ignite a solid energetic material by a laser pulse of a Gaussian shape is derived. It is shown that, if the laser beam radius is $r_0 \leq \alpha^{-1}$ (α is the light absorbance), ignition of condensed energetic materials with a light pulse depends on the beam size. The heat conduction equation allowing for laser beam absorption is solved. The calculation results agree with the derived criterion.

The condition for solid fuel and explosives' ignition depends on the laser pulse parameters – energy density, duration and the diameter of a light beam [1–5].

We numerically solved a three-dimensional nonlinear equation of heat conductivity

$$c\rho\frac{\partial T}{\partial t} = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \alpha I \exp(-\alpha z) + qK_0 \exp\left(-\frac{E}{kT}\right), \qquad (1)$$

with the following initial $T(x, y, z, 0) = T_0$ and boundary conditions

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = \left.\frac{\partial T}{\partial x}\right|_{x=L_x} = 0, \quad \left.\frac{\partial T}{\partial y}\right|_{y=0} = \left.\frac{\partial T}{\partial y}\right|_{y=L_y} = 0,$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = \left.\frac{\partial T}{\partial z}\right|_{z=L_z} = 0, \qquad (2)$$

where L_x, L_y and L_z are the transverse and longitudinal sample sizes; T is the temperature, T_0 is the initial temperature; λ, c are the heat conductivity coefficient and heat capacity of the material; ρ is the density of the material; q, K_0, E are the reaction heat effect and the pre-exponential factor of the decomposition reaction and activation energy of the decomposition rate, respectively; k is the Boltzman constant; α is the absorption coefficient; L_z is the laser radiation intensity.

External heat sink was not taken into account. We suppose that the laser pulse duration and ignition delay time are much less than the characteristic time of external heat sink.

The intensity distribution over the beam cross section is Gaussian

$$I(x, y, t) = I_0(t)\exp\left[-\frac{x^2 + y^2}{r_0^2}\right],$$

where r_0 is the characteristic radius of the beam; I_0 is the energy flow density at the beam center, defined by the following expression

$$I_0(t) = \frac{W}{6\tau_m}\left(\frac{4t}{\tau_m}\right)^4 \exp\left(-\frac{4t}{\tau_m}\right).$$

Here τ_m is the duration of the first front of the pulse concerning the pulse duration, measured on the half-height by the expression τ_i=1,19τ_m; W is the energy density of the laser pulse. A laser radiation beam is normally incident on the solid surface, parallel to the z axis. The coordinate origin is chosen in the point of intersection of the beam axis and the surface plane.

The numerical solution to a three-dimensional equation of heat conductivity with a nonlinear heat source (1) and initial and boundary conditions (2) was carried out by means of subdividing the problem according to space variables. The difference net with the x, y and z variables was divided according to the control volume method. The resulting system of the implicit difference equations according to the corresponding space directions was solved by the sweep method. The difference net was chosen as irregular. Arrhenius nonlinearity was linearized for every time step by means of Frank-Kamenetsky's transformation.

The calculations were carried out with T_0=300K and the following thermo-physical and kinetic parameters of an explosive: c=1,47 J/(gMK), λ=2,35·10^{-3} W/(cmMK), ρ=1,6 g/cm^3, qK_0=8,04·10^{16} J/(cm^3·s), E=1,43 eV and absorption coefficient α=10^2 cm^{-1}.

The results of the numerical solution to the problem of condensed material ignition are submitted in Fig. 1 – Fig. 4. The calculations were carried out at different energy densities W, the light beam radius r_0 and laser pulse duration τ_i.

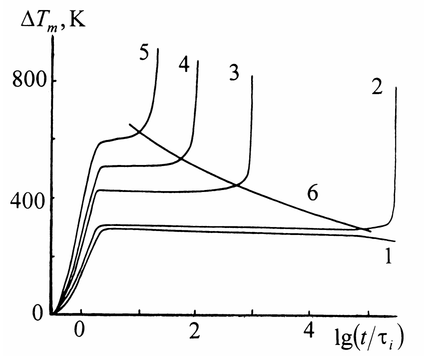

Fig. 1. Temperature ΔT_m vs. time at τ_i=10^{-8} sec, r_0=10^{-1} cm and W = 7 (1), 7,3 (2), 10 (3), 12 (4) and 14 J/cm^2 (5); 6 – the curve crossing curves 2 – 5 in the points t=τ

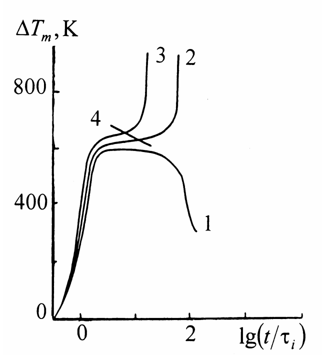

Fig. 2. Temperature ΔT_m vs. time at τ_i=10^{-8} sec, r_0=10^{-4} cm and W=14 (1), 14,5 (2) and 15 J/cm^2 (3); 4 – the curve crossing curves 2, 3 in the points t=τ

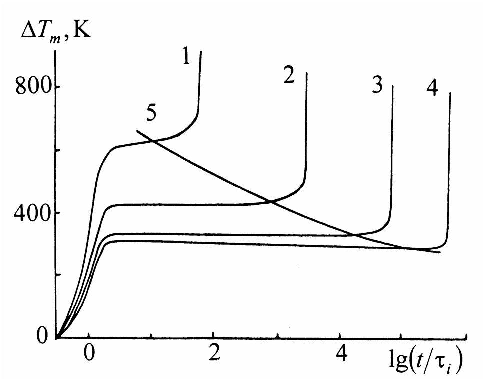

Fig. 3. Temperature ΔT_m vs. time at τ_i=10^{-8} in the vicinity of the ignition threshold: r_0=10^{-4} cm, W=14,5 J/cm^2 (1); r_0=10^{-3} cm, W=10,0 J/cm^2 (2); r_0=10^{-2} cm, W=7,8 J/cm^2 (3); r_0=10^{-1} cm, W=7,2 J/cm^2 (4); 5 – the curve crossing curves 1 – 4 in the points t=τ

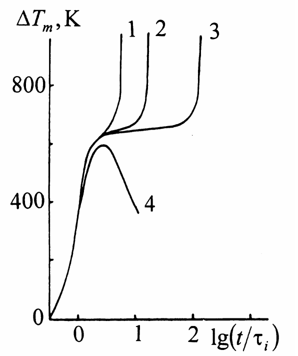

Fig. 4. Effect of the duration of a laser pulse on temperature ΔT_m vs. time at r_0=10^{-4} cm and W=15 J/cm^2: τ_i=10^{-9} (1), 10^{-8} (2), 10^{-7} (3), $2\cdot 10^{-7}$ (4)

The calculations showed that the effect of a wide beam (r_0>>α^{-1}) made the critical density of condensed material ignition energy much less than that of a narrow beam (r_0<<α^{-1}). However, for the narrow beam the ignition delay time τ is less, because decrease in r_0 results in increase in ignition critical energy and, correspondingly, in temperature increase of the surface after the laser pulse ends, which decreases adiabatic warming-up time of the reaction zone of solids.

The results of the numerical calculations of ignition energy critical density at the laser pulse duration $\tau_i = 3\cdot 10^{-8}$ s are given in Table.

Table. The effect of the light beam radius on the critical density of ignition energy of a model explosive

r_0, cm	10^{-1}	10^{-2}	10^{-3}	10^{-4}
W, J/cm^2 (calculation)	7,2	7,8	10,0	14,5
W, J/cm^2 (theory)	7,11	7,25	9,28	13,5

In our work we derived the analytical expression for the critical density of ignition energy depending on the beam diameter of a short laser pulse. Extending the criterion of solid materials' ignition [6] to a two-dimensional case, we can write it in the cylindrical coordinate system with account of the problem symmetry

$$2\pi q K_0 \int_0^\infty \int_0^\infty \exp\left(-\frac{E}{kT}\right) r dr dz =$$

$$= -\lambda\pi r_1^2 \left.\frac{\partial T_1}{\partial z}\right|_{r=0} - 2\lambda\pi r_1 z_1 \left.\frac{\partial T_1}{\partial r}\right|_{z=0}, \qquad (3)$$

where T_1 is the temperature on the reaction volume boundary determined from the condition that the reaction rate on the reaction volume boundary is e times as low as that at the origin point on the surface. In the case of short laser pulse absorption, the temperature distribution in the sample after the pulse ends, is determined by the absorbed energy distribution

$$T(r,z) = T_0 + \left(\frac{\alpha W}{c\rho}\right)\exp\left(-\alpha z - \frac{r^2}{r_0^2}\right). \qquad (4)$$

To calculate the integral in expression (3) we take the Taylor series for the function $T(r,z)$ in the vicinity of the ignition temperature $T_m = T(0,0)$, using Frank-Kamenetsky's transformation

$$\frac{1}{T} = \frac{1}{T_m}\left(1 - \frac{z}{T_m}\frac{\partial T_m}{\partial z} - \frac{r^2}{2T_m}\frac{\partial^2 T_m}{\partial r^2}\right). \qquad (5)$$

With account of expressions (4) and (5), after integrating and differentiating of expression (3) with respect to r and z, we obtain the criterion for condensed material ignition depending on the laser beam radius if $\gamma << 1$

$$z_1 q K_0 \exp\left(-\frac{E}{kT_m}\right) = \lambda\Delta T_m\left(\alpha + \frac{4z_1}{r_0^2}\right). \qquad (6)$$

Here $\gamma = kT_m/E$, $\Delta T_m = \alpha W/c\rho$, and the expressions for characteristic sizes of the reaction volume $v_1 = \pi r_1^2 z_1$ are the following [7, 8]

$$z_1 = \alpha^{-1} \ln\left(\frac{1+\gamma}{1-\gamma T_0/\Delta T_m}\right),$$

$$r_1 = r_0\left[\ln\left(\frac{1+\gamma}{1-\gamma T_0/\Delta T_m}\right)\right]^{1/2}.$$

The results of calculating the critical ignition energy of a model explosive are given in Table. As seen, the results of the ignition energy critical density agree with the results of a numerical solution to the heat conductivity equation.

In our work we obtained the expression for the ignition delay time of a condensed material after the laser pulse ends from the solution to the equation for the temperature change in the reaction volume

$$\frac{d\theta}{d\xi} = \exp\theta - \frac{t_{ad}}{t_1} \quad (7)$$

if the exothermal reaction rate after the induction period increases e times as much. Here

$$\theta = \frac{\Delta T}{\gamma T_m},\ \xi = \frac{t}{t_{ad}}$$

are non-dimensional temperature and time; t_{ad}, t_1 are the adiabatic warming-up time and the characteristic time of thermal relaxation of the reaction volume determined by the following expressions

$$t_{ad} = \frac{c\rho k T_m^2}{qK_0 E}\exp\left(\frac{E}{kT_m}\right),$$

$$t_1^{-1} = a\left(\frac{4}{r_1^2} + \frac{1}{z_1^2}\right).$$

Here $a = \lambda/c\rho$ is the heat conductivity coefficient. With account of new variables the ignition criterion takes the form of

$$\frac{t_{ad}}{t_1} = 1,$$

that is, the adiabatic period of induction in the critical point is equal to the characteristic time of thermal relaxation of the reaction volume. After solution (7) we obtain

$$\frac{t_1}{t_{ad}}\left[-\theta + \ln\left(\frac{\exp\theta - t_{ad}/t_1}{1-t_{ad}/t_1}\right)\right] = \xi,$$

and the ignition delay time is determined by the equation

$$\tau = t_1 \ln\left(\frac{1-t_{ad}/et_1}{1-t_{ad}/t_1}\right), \quad (8)$$

which at $t_{ad}/t_1 << 1$ takes the form: $\tau \approx 0,632 t_{ad}$. Therefore, delay time no longer depends on the light beam diameter when energy density exceeds the critical value. The ignition delay time for gunpowder vs. the characteristic radius and energy density of a laser beam are calculated in terms of formula (8): curve 6 in Fig. 1; curve 4 in Fig. 2; curve 5 in Fig. 3. These curves cross the kinetic curves $\Delta T_m = f(t/\tau_i)$ in the points $t = \tau$. As seen from Fig. 1 – Fig. 3, the ignition delay time calculated in terms of formula (8) is in a satisfactory agreement with the characteristic sharp bend on the curves of the temperature change of the surface with time.

The results of the calculations in terms of formula (8) of solid material ignition delay time dependence on the laser pulse radius at critical energy densities are presented in Fig. 5. As seen, the delay time decreases with decreasing the beam radius.

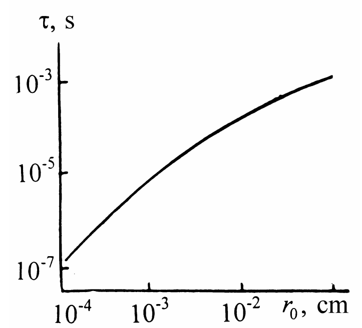

Fig. 5. Ignition delay time of a solid material τ at a critical energy density of a laser pulse vs. the radius of a light beam

The experimental data [5] on initiating of lead azide with the pulse team are presented in Fig. 6: delay time decreases with decreasing a laser pulse radius (curve 1), and the critical energy density increases (curve 2), which is in a qualitative agreement with the results of our work.

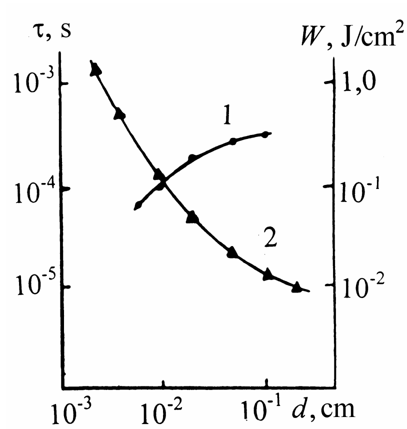

Fig. 6. Delay time for initiating of lead azide τ (1) and the critical energy density W (2) vs. the diameter of a light beam when effected by a pulse team [4]

Not only the sizes of a light beam and energy density but also the light pulse duration, influence the ignition condition of a solid material. At the pulse duration $\tau_i \ge t_1$, the critical ignition energy of an explosive increases.

In the case of the absorption factor depending on the temperature, the criterion of ignition (6) takes the form of ($\gamma << 1$)

$$z_1 q K_0 \exp\left(-\frac{E}{kT_m}\right) =$$

$$= \lambda \Delta T_m \left\{ \frac{\alpha}{1-(1-\alpha z_1)\Delta T_m \partial \ln \alpha / \partial T} + \right.$$

$$\left. + \frac{4 z_1}{r_0^2 (1 - \Delta T_m \partial \ln \alpha / \partial T)} \right\}.$$

At $\partial \ln \alpha / \partial T > 0$ both the critical temperature of ignition and the critical density of energy of a laser pulse decrease.

"Cooling" of the reactionary volume occurs not only owing to the diffusion of heat into the bulk of the explosive but also owing to evaporation. In the case of evaporation the criterion of ignition (6) takes the form of

$$z_1 q K_0 \exp\left(-\frac{E}{kT_m}\right) = \lambda \Delta T_m \left(\alpha + \frac{4 z_1}{r_0^2}\right) + L J_m, \qquad (9)$$

where L – heat of evaporation, J_m – a flow of particles evaporating from the surface. The flow $J_m = J_0 \exp(-L / kT_m)$. The results of the calculation of critical thermal energy flows vs. the radius of a laser beam are given in Fig. 7. Thermal flows were calculated in terms of the formulas

$$P_1 = z_1 q K_0 \exp\left(-\frac{E}{kT_m}\right), \quad P_2 = \lambda \alpha \Delta T_m,$$

$$P_3 = 4\lambda \frac{z_1 \Delta T_m}{r_0^2}, \qquad P_4 = L J_m.$$

The calculations were made at L=1,4 eV, J_0=10^{31} cm^{-2}c^{-1}.

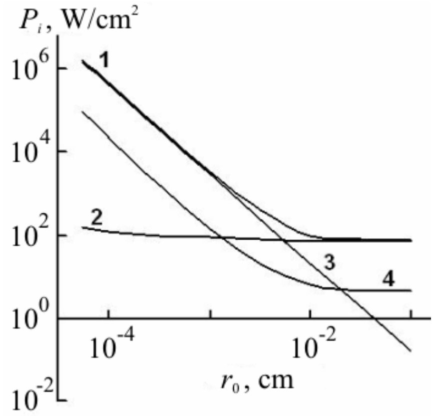

Fig. 7. Critical thermal flows of energy vs. the radius of a laser beam: 1 – P_1, 2 – P_2, 3 – P_3, 4 – P_4

As seen from Fig. 7, for a wide beam, the critical energy of ignition of an explosive depends on a normal component of the heat sink P_2, and for a narrow beam – a radial component P_3 and evaporation P_4.

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