Monte-Carlo Simulation of Passage of Photons in Semiinfinite Diffusely Scattering Medium

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Abstract – The paper presents the results of processes simulation and light-scattering mechanism into diffusing scattering medium by using Monte Carlo method. The empirical ratio to connect the spatial light exposure in volume of scattering medium with the size of a laser beam on a sample surface and its factor of diffusing reflection are obtained.

The study of optical radiation propagation in light-scattering medium is the important scientific problem. Recently interest in this problem has appreciably grown that is substantially connected to development of optics of diffusely scattering medium (DSM) that inorganic medium and biological tissue concern. In such medium measurement of spatial distribution intensity of light and their optical characteristics is necessary, for example, for determining an optimum dosage at diagnostics and biological tissue and objects [1].

The approach based on theory radiation transfer is the most applicable for determining of light intensity inside and outside scattering medium. From existing practically significant methods of solution of the radiation transport equation the Monte Carlo method of computational modeling [2] is the most universal. It can be used for any geometry and for any medium (homogeneous and heterogeneous).

Traditionally in general form the radiation transfer is described equation which can be written as follows [3]:

$$\frac{\partial}{\partial S} I(\bar{r},\bar{s},t) + t_2 \frac{\partial}{\partial t} I(\bar{r},\bar{s},t) = -\mu_t I(\bar{r},\bar{s},t) + \frac{\mu_s}{4\pi} \int_{4\pi} \left[\int_{-\infty}^{t} I(\bar{r},\bar{s}',t') f(t,t') dt' \right] p(\bar{s},\bar{s}') d\Omega' \quad (1),$$

where $I(\bar{r},\bar{s},t)$ - beam intensity in a point r in a direction s, W· m^{-2}·sr^{-1};

$p(\bar{s},\bar{s}')$ - phase function of dispersion;

μ_s – dissipation factor (magnitude describing average quantity of elastic dispersion acts in which photon participates at run on length unit);

$\mu_t = \mu_s + \mu_a$ – coefficient extinction;

μ_a – absorption factor (magnitude reverse of the distance in which the beam is weakened due to absorption in e time);

$d\Omega'$ – unitary solid angle in direction s′;

$\mu_s/\mu_t = \Delta$ – reflectance unitary diffuser;

f(t,t′) – describes the time deformation δ – a figurative pulse after a single dispersion act.

Beam intensity $I(\bar{r},\bar{s},t)$ contains two components: weakened falling radiation and diffused one.

Phase function $p(\bar{s},\bar{s}')$ describes dispersive characteristics of medium and represents the density probability function of dispersion in a direction s' a photon moving in a direction s, i.e. it characterizes the elementary dispersion act. If dispersion is symmetric concerning a falling wave direction the phase function depends only on angle θ between directions s and s', i.e. $p(\bar{s},\bar{s}') = p(\theta)$ (Fig. 1).

Integro-differential equation (1) is complex for analysis of light passage in diffusing medium. Therefore frequently it becomes simpler by representation of decision as spherical harmonics. Such results simplification leads to the system of connected differential equations in partial derivatives which can be reduced to the unique equation of diffusive type.

Strict solution of transport equation (1) can be received by the method of discrete ordinates when this equation will be transformed to the matrix differential equation for illumination on many discrete directions. This solution approximates to exact at increase in number of angles. It is possible also the illumination expands into series on spherical harmonics with separating of transport equation into components of spherical harmonics. At sufficient number of spherical harmonics such way also leads to exact solution.

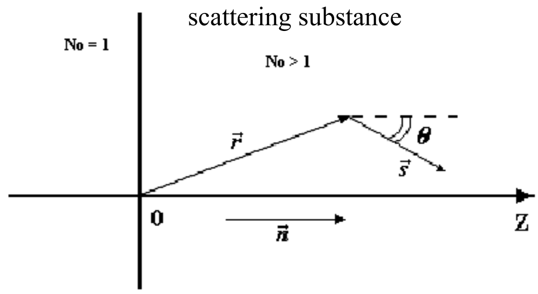

Fig. 1. Scheme of photons transmission in DSM

These methods [4,5,6] give satisfactory fit with exact solutions of transport equation but its opportunities also are limited to definition of integrated characteristics of field in case of flat geometry (unlimited beam sizes) and isotropic dispersion. In most practical cases the radiator geometry must be taken account and dispersion is inadmissible to count isotropic. Therefore an imitating modeling method of radiation transport process – Monte Carlo method is on special importance.

The totality expedients allowing to receive necessary solutions by means repeated of random trials is understood as a Monte Carlo method. Estimations of sought magnitude are obtained by statistical way. This method does not assume the solution of non-stationary equation of radiation transport theory (1).

Application this method is based on use macroscopic optical characteristics of medium which are assumed homogeneous within researched area of medium. Developed algorithm allows taking into account some layers of medium with various optical characteristics, final size of falling beam, light reflection from the frontiers of layers. In program of the Monte Carlo method is realized as repeated tests of photon transport inside medium.

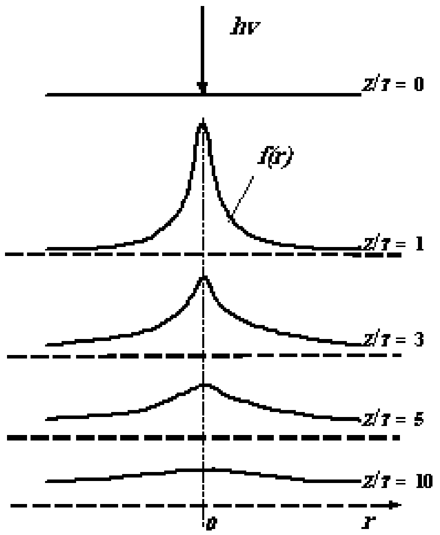

Fig. 2. A light mode in volume disseminating medium (n_0 = 1,85) at dot directed radiator. $\tau = 1/(\mu_a + \mu_s)$ – average track length (optical thickness). Curves *f (r)* – essence of point degradation function

For getting an image of light-diffuse it is necessary to follow up path of large quantity of photons including acts of absorption, dispersion, reflection from frontiers of mediums and yield out scattering sample. Essence of a method consists in consecutive modeling of photons trajectories with help of determination of interaction coordinates of particles with medium on the moving direction and free run. The photon path is tested with the help of random numbers located in the range [0 – 1] and determining probability of realization process. At absorption photon or output one abroad medium (photon disappearance) the trajectory breaks and trajectory new is simulated.

In essence the direct modeling of photons trajectories does not differ from modeling of neutrons or gamma-quantum trajectories which is used for solution of nuclear physics problems [7,8]. Distinctions are observed only in processing of account results.

For modeling passage of a photon through DSM the half-infinite layer has been divided on equal cells in size of τ on x, y, z. In Fig. 2 the typical relative distribution of spatial illumination on various optical depths, where $\tau_z = (\mu_a+\mu_s)Z$, giving general characteristic of the light mode inside a layer for directed dot radiator on DSM surface is shown. Separate curves represent essence of a degradation function of a point on the chosen optical depth. For determination of size of spatial illumination the summation of "traces" of straight and scattered photons which have got in elementary volume of medium from all possible directions was carried out. With the purpose increase in statistical reliability of summation results of scattered photons trajectories the average of results in volume in the size τ^3 was carried out.

As initial parameters for modeling the following data were used: indefinitely narrow beam (paraxial, falling in one point) directed perpendicularly of DSM surfaces with refractive factor of medium n_0 = 1,85 (heavy metal azides).

Diffuse reflection factor ρ_d was determined with the help very simple algorithm:

$$\rho_\partial = N_B / N \quad (2),$$

where N_B – number ("survived") photons returned back from the medium. With increase reflectance of unitary dispersion the illumination inside a layer sharply grows, and this increase is especially appreciable on the large depth.

Table 1. Reflection factor ρ_∞ from flat layer of DSM with refractive factor n_0 = 1,5, calculated by various ways. (χ = 1)

Δ	Analytical calculation [9]	Monte Carlo	Deviation in %
0,9995	-	0,8797	-
0,999	0,841	0,8307	1,012
0,99	0,602	0,5852	1,029
0,9	0,260	0,2305	1,128
0,8	0,172	0,1423	1,209

The results of comparisons (see Table 1) specify that accuracy of the method at the set photons number is satisfactory. Divergences in the values are essential only in case of small survival rate of a photon (Δ = 0,8).

Relationship coefficient between volumetric (spatial) q_n and superficial q_0 illuminations calculate with the help of algorithm:

$$F(y,z) = \frac{q_n(y,z)}{q_0} = \frac{N_k(y,z)}{N} + A\exp(-\frac{z}{\tau}+1) \quad (3),$$

where N – photons number falling on an single plane τ^2 ; N_k – number of acts of dispersion and absorption in the chosen single volume τ^3 on depth z from a substance surface; A = 1 if chosen volume is in the field of falling beam in the size d = 2r, and A = 0, if outside the volume. The calculations show that distribution of illumination in generally case (medium with appreciable absorption, limited beam sizes, $\Delta < 1$, $r < \infty$) into the volume of half-infinite layer depends not simple on many parameters (Δ, r, χ, n_0 a corner inclination dragging beam).

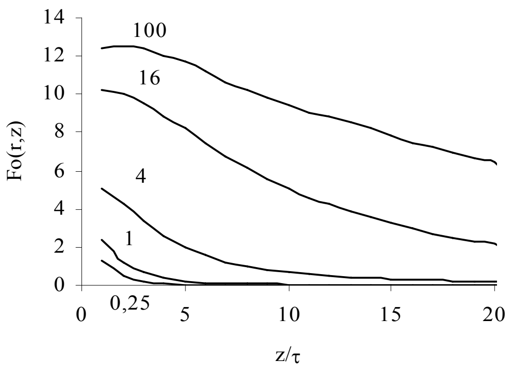

Fig. 3. Connection between spatial illumination q in object depth and illumination q_0 ($q/q_0 = F_0$) created by a laser beam on DSM surface.
Figures specify the size of a laser beam on a medium surface (in terms r/τ, where τ – optical thickness layer)

In Fig. 3 the distribution of spatial illumination on depth of DSM layer for limited beam sizes at normal falling beams is resulted. Strong dependence of illumination inside volume from the size r of a light beam on a surface is observed, namely:

– Occurrence of a illumination maximum on some depth in the field of a transitive mode which amplitude grows, and position is shifted in depth r;
– Sharp recession of illumination in a deep mode with reduction of r.

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