Spatial Density Distribution of a "Squeezed" High-Current Electron Beam[1]

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Abstract – Based on the laws of conservation of energy and z-component of the field and particle momentum, it has been shown that under certain conditions a slightly inhomogeneous electron beam "squeezed" by a virtual cathode in a two-section drift tube exhibits stable spatial density distribution. The period and amplitude of space variations in the electron beam density have been calculated. Numerical simulation has demonstrated that in a smooth drift tube the spatial structure relaxes to the stationary state. A physical interpretation of the phenomenon has been given using the model of "potential" functions for electrons of the beam.

1. Introduction

It is well known that when an electron beam is injected into a two-section drift tube a virtual cathode (VC) develops at the tube junction under certain conditions and, when the injected current reaches a certain critical value, called the transition current (I_{Tr}), the VC streams toward the injection region, leaving the electron beam in a single-flux "squeezed" state behind [1-4].

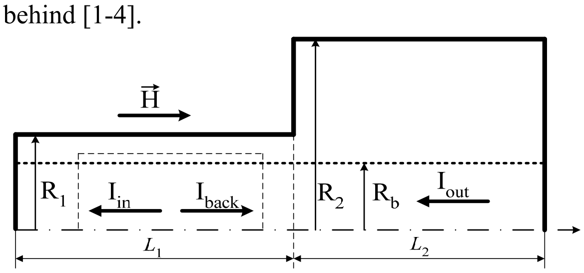

Fig. 1. Schematic of the two-section drift tube bounded by foils. R_1, R_2, L_1, L_2– radii and lengths of the section, R_b – beam radius; the dashed line stands for the region in the narrower section where the field distribution is independent of the z coordinate.

A detailed study of this phenomenon by numerical simulation has disclosed that under certain conditions a quasistationary spatially periodic structure (space variations in electron beam density and relativistic factor) with a certain period and amplitude may form behind the VC on the right. Neither these periodic structures nor the ways of their formation have been studied so far. In [5] theoretical investigation evidenced the possibility that periodic solutions for the relativistic factor and for the electron density in a smooth drift channel may exist, but the lack of explanation for the "squeezed" state and for the patterns of its realization by that moment did not allow research into these structures and solutions.

2. Theory

Consider a two-section drift tube with faces, each covered with a foil transparent for electrons (Fig.1). The tube has the electrostatic potential of an anode. A thin annular monoenergetic electron beam of radius R_b is injected into this system from the left. Let the whole system be placed in a strong longitudinal guiding magnetic field of strength $\vec{H}$ parallel to the tube axis. The magnetic field strength is so large that the Larmor radius of an electron is much smaller than the beam diameter.

For an electron beam in a smooth homogeneous tube section, assuming the presence of longitudinal electric fields, we can write [5]:

$$\left(\gamma_b'\right)^2+\frac{2\gamma_b\left(2\Gamma-\gamma_b\right)}{R_b^2\ln\left(R_1/R_b\right)}-\frac{8}{R_b^2}\frac{I\left(\gamma_b-1\right)^{1/2}}{I_0}=Const \quad (1)$$

where $\gamma_b=1\Big/\sqrt{1-v_b^2/c^2}$ is the relativistic factor of electrons in the beam, Γ is the relativistic factor corresponding to the total voltage and $I_0 = mc^3/e \approx 17$ kA.

Note that equation (1) coincides, to an accuracy of constant, with the law of conservation of the z-component of the field and particle momentum for an electron beam transported in a smooth homogeneous drift tube, without disturbance [1].

Introducing the notation:

$$\begin{aligned}F\left(\gamma\right)=&\frac{2}{R_b^2\ln\left(R_0/R_b\right)}\gamma_b\left(2\Gamma-\gamma_b\right)-\\&-\frac{4}{R_b^2\ln\left(R_0/R_b\right)}\kappa\left(\gamma_b-1\right)^{1/2}\end{aligned}, \quad (2)$$

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where

$$\kappa = \left(\Gamma - \gamma_b\right)\sqrt{\gamma_b^2 - 1}\Big/\gamma_b\,. \qquad (3)$$

and expressing equation (1) in these terms gives:

$$\left(\gamma_b'\right)^2 + F\left(\gamma\right) = Const\,. \qquad (4)$$

It is known [6] that an equation of type (4) describes one-dimensional motion of a particle in the field of generalized potential $F(\gamma)$ without external driving force. The character of the motion for particles of different energies is determined by the type of a "potential" function. Figure 2 shows "potential" functions for different values of the parameter κ (the dimensionless electron beam current). For convenience, the functions are normalized to the "potential" field energy equal to (2Γ-1)/ $R_b^2 \ln\left(R_1/R_b\right)$, i.e., for $\gamma_b = 1$. At currents lower than the limiting transport current ($\kappa < \kappa_{\lim} = 1.12$), the function $F(\gamma)$ is described by a double-humped curve, and the extremum of the function are found from relation (3). The smallest value of γ_b corresponds to an electron beam in the "squeezed" state. At beam currents $\kappa \rightarrow \kappa_{\lim}$, these extremum are degenerated to a point (point A) and $\gamma_b = \Gamma^{1/3}$. At beam currents $\kappa > \kappa_{\lim}$, the curve has no singular point.

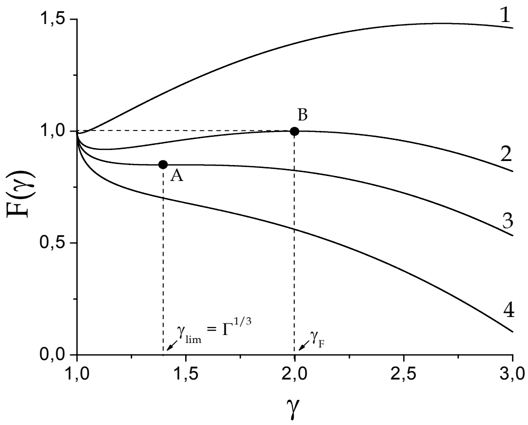

Fig. 2. "Potential" curves for different beam currents κ: 1) κ = 0.3, 2) κ = 0.866 = κ_F and corresponds to the Fedosov current I_F, 3) κ = 1.12 = $\kappa_{\lim}$ and corresponds to the limiting transport current I_{lim} for the given geometry, 4) κ = 1.5. Γ = 3.

In the general case, we should integrate equation (4) by calculating the constant in homogeneous drift regions free of the longitudinal component of the electric field $\partial\gamma_b/\partial z = 0$. The exact periodic solutions for the "squeezed" electron beam can be obtained also by linearizing equation (1), given small deviations of the electron energy from the mean ($\gamma = \overline{\gamma} + \tilde{\gamma}$):

$$\partial^2\tilde{\gamma}/\partial z^2 - \omega^2\tilde{\gamma} = 0\,, \qquad (5)$$

where $\omega^2 = 2\left(\overline{\gamma}^3 - \Gamma\right)\Big/\left(\overline{\gamma}\left(\overline{\gamma}^2 - 1\right)R_b \ln\left(R_0/R_b\right)\right)$ (see [5]).

3. Simulation of the spatial beam structure

The density distribution of the squeezed electron beam has been calculated by simulating the transport of a thin annular electron beam in a two-section drift tube with the use of the PIC code KARAT [7].

The geometry of the drift tube used in the calculations is shown schematically in Fig.1.

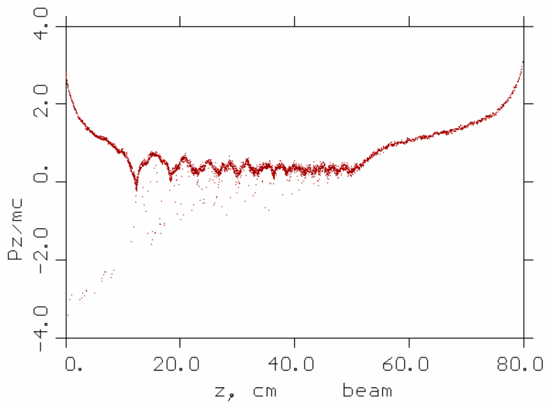

Fig. 3. Phase-plane portrait of the electron beam in the two-section drift tube when the squeezed beam assumes anomalous spatial density distribution and relaxes to the stationary state. Calculations by the PIC code KARAT. R_b = 2 cm, R_0 = 4 cm, L_1 = 50 cm, R_2= 8 cm, L_2 = 30 cm. Γ = 3.

In all calculations, the radius of the narrow drift section and the beam radius remained constant (R_1 = 4cm, R_b = 2 cm, L_1 = 50), whereas the radius of the wide section was varied from 4.1 cm to 20 cm, depending on the requisite transmitted current. An electron beam with initial energy U = 1.5 MeV was injected through the left foil of the drift tube. The drift tube was placed in a homogeneous longitudinal guiding magnetic field of strength H = 800kOe. Figure 3 shows the calculated phase portraits of the "squeezed" electron beam when the latter acquires anomalous spatial density distribution and fills completely the narrow section of the drift tube. It can be seen from this figure that the spatial fluctuations behind the VC on the right decreases in amplitude as the VC shifts. Calculations have shown that this decrease owes to variation in the average electron density and average relativistic factor behind the VC. After a time (of order 10 ns), the density distribution relaxes to the stationary state for an infinite smooth channel with a certain transmitted current [3]. In the example given in Fig. 3, the current risetime is 50 ns for an injected current of 20 kA. For comparison, the same system has been calculated with longer current risetimes (up to ≥150 ns). The simulation has shown that the spatial structure, in this case, does not occur that is presumably associated with a slow VC velocity.

Figure 4 shows the γ dependence of the period of the spatial structure for the squeezed electron beam.

This dependence was calculated by direct integration of equation (1) for a specified value of the "potential" function. It is seen from this figure that when the deviation from the mean $\overline{\gamma}$ is small, the harmonic approximation agrees closely with the results of numerical simulation and direct integration of equation (1). With large deviations, the period of spatial fluctuations and the harmonic approximation differ considerably due to the potential well asymmetry. However, the explicit nonstationarity of the processes occurring in this case does not allow numerical estimation of the electron density distribution behind the VC.

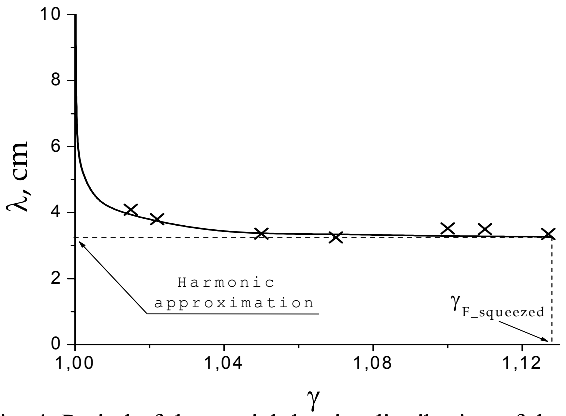

Fig. 4. Period of the spatial density distribution of the squeezed electron beam versus the relativistic factor of electrons for the current $\kappa = \kappa_F$. Solid curve stands for the numerical solution of the equation with specified boundary conditions and the dashed line for the period calculated in the harmonic approximation. × - calculations by the PIC code KARAT. Γ = 3.

4. Discussion of the results

In the quasistationary case, the shift of the VC causes the beam behind the VC on the right to pass into the squeezed state which, according to the laws of conservation, features an increase in electron density. Figure 5 shows two potential curves for the beam to the left (curve 2) and to the right (curve 1) of the VC. Since the transmitted current can be taken near-constant (which it is not because of the current risetime), the passage of the beam into the squeezed state corresponds to the segment AB, with the value of the potential function *F*(γ) remained constant. Reasoning from the notion of "potential" functions, the beam in its passage to the squeezed state, may be thought as occurring in the multitude of potential wells between the points A and B. In this case, the distribution γ(z) will take the form of fluctuations. Computer simulation has shown that the amplitude and period of spatial fluctuations of the electron density behind the VC grow smaller with time. On this basis it can be stated that the amplitude and period of spatial fluctuations depend on the VC velocity and increase with increasing VC velocity. Thus, in this work the spatial density distributions of the squeezed electron beam have been originally obtained by numerical simulation and the interpretation of the phenomenon has been given based on "potential" functions for a slightly inhomogeneous electron beam.

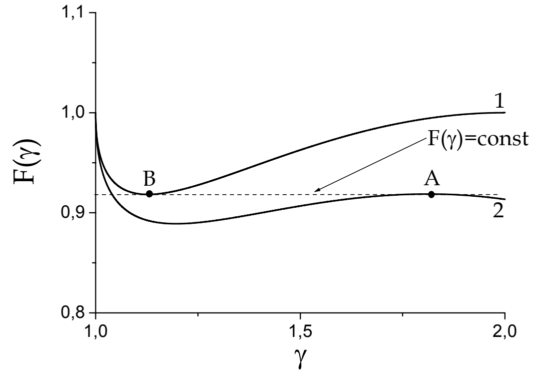

Fig. 5. "Potential" functions of the electron beam for the currents κ = 0.866 and κ = 0.995 normalized to 2(Γ-1).

References

[1] S. Ya. Belomyttsev, A. A. Grishkov, S. D. Korovin, and V. V. Ryzhov, Pis'ma Zh. Tekh. Fiz. **29** (16), 16 (2003) [Tech. Phys. Lett. **29**, 666 (2003)].
[2] S. Ya. Belomyttsev, A. A. Grishkov, S. D. Korovin, V. V. Ryzhov, Laser and Particle Beam, **21**/4, 561, (2003)
[3] A. A. Grishkov, S. Ya. Belomyttsev, S. D. Korovin, V. V. Ryzhov, Pis'ma Zh. Tekh. Fiz. **29**,22 (2003) [Tech. Phys. Lett. 29, 666 (2003)].
[4] Ignatov A.M., Tarakanov V.P. Phys. Plasmas, Vol.1, №3, p. 741 (1994).
[5] N.F.Kovalev, M.I. Fuks, On the stationary states of thin-wall high-current relativistic electron beams, Preprint 13, Gorky (1980).
[6] L.D. Landau, E.M. Lifshitz, Classical Mechanics Nauka, Moscow, 1988.
[7] V.P. Tarakanov. User's manual for code Karat. Berkley Research Associate. Inc, Springfield, VA (1992).