Ion Emission from Plasma

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Abstract – The analysis carried out by Bohm on ion emission from plasma contains rough contradictions. To take off these contradictions is possible if to consider generation of ions in the presheath, and not only their acceleration. The joint analysis of the equations of motion and continuity, carried out in this paper, allows to obtain formulas for ion emission current density, suitable in collision and collisionless modes.

1. Introduction

The Bohm formula [1] is usually used for an estimation of current density of ion emission from plasma

$$j = 0{,}61en_0\sqrt{\frac{kT_e}{M}}\,, \qquad (1)$$

where e is the elementary charge, n_0 is the plasma density, k is the Boltzmann constant, T_e is the electron temperature, M is the ion mass. During the carried out analysis Bohm had shown, that on plasma - sheath border the energy of ions should satisfy to the following condition

$$e\varphi_0 \geq \frac{kT_e}{2}\,, \qquad (2)$$

and the average speed of ions on this border v_b should be not less, accordingly, than $(kT_e/M)^{1/2}$. This condition is known as the Bohm criterion. To explain, how the ions get such speed, it was offered to share plasma into area of the undisturbed plasma with concentration n_0, in which the electrical field is absent, and region of the disturbed plasma, in which condition of quasineutrality is satisfied, but nevertheless there is an electrical field accelerating ions to the plasma - sheath border. This region of the disturbed plasma was named by Bohm as a presheath. If to accept, that in the presheath the difference of potential $\varphi_0 = kT_e/2$ is concentrated, accelerating ions up to speed $(kT_e/M)^{1/2}$, and electrons are distributed according to Boltzmann formula, then the following relation for concentration of plasma on presheath – sheath border is fulfilled

$$n_b = n_0\, exp\left(-\frac{e\varphi_0}{kT_e}\right) = n_0\, exp(-\frac{1}{2}) = 0{,}61n_0\,, \quad (3)$$

Then, after multiplication of the received value to a charge and the speed of an ion, one can obtain the above mentioned Bohm formula for ion emission current density.

The carried out analysis contains a rough contradiction. If one consider, that there is only acceleration of ions at absence of processes of ionization in the presheath, then from the continuity equation we receive, that the same current should proceed on border between the undisturbed plasma and presheath. So we obtain the following relation

$$j=en_bv_b=en_0v_0 \qquad (4)$$

where v_0 is the speed of ions on the undisturbed plasma – preasheath border. So in the undisturbed plasma, where the electrical field is absent, the ions, nevertheless, will be accelerated up to speed $v_0 = j/(en_0) = 0{,}61\sqrt{kT_e/M}$. Besides the ions should get into presheath with this speed, while it was accepted, that the potential fall in presheath should provide acceleration of ions up to $\sqrt{kT_e/M}$ from *0*. To take off these contradictions is possible if to consider, that not only acceleration of ions occurs in the presheath, but also their generation. So one should carry out the joint analysis of the equations of motion and continuity.

2. Collisionless mode

Let's write down the continuity equation

$$\frac{d(nv)}{dx} = G(x)\,, \qquad (5)$$

where G is the number of ions generated by an external source in unit of time in unit of volume, or $G = \nu_i n$, if ionization is carried out by plasma electrons. Neglecting collisions of ions with atoms we shall write down the equation of motion in the following kind

$$Mv\frac{dv}{dx} = -e\frac{d\varphi}{dx} - \frac{kT_i}{n}\frac{dn}{dx} - \frac{G}{n}Mv\,. \quad (6)$$

As for electrons we shall use the Boltzmann law

$$n = n_0\, exp\left(\frac{e\varphi}{kT_e}\right). \qquad (7)$$

Solving the written down system of equations we shall receive for speed of ions the following expression

$$v = v_s\sqrt{\frac{n_0}{n} - 1}\,, \tag{8}$$

where the designation

$$v_s = \sqrt{\frac{k(T_e + T_i)}{M}} \tag{9}$$

is entered. Substituting the received relation in the continuity equation one can receive after differentiation the following expression for derivative of concentration

$$\frac{dn}{dx} = \frac{G\sqrt{\frac{n_0}{n} - 1}}{v_s\left(\frac{n_0}{2n} - 1\right)}\,. \tag{10}$$

The denominator becomes equal 0 at $n=n_0/2$, and derivative of concentration, so also derivative of potential address in infinity, i.e. there is an infringement of quasineutrality. Thus $n_b=n_0/2$ on presheath – sheath border, and $v=v_s$ at this value of concentration. Then we obtain for ion emission current density

$$j = \frac{en_0}{2}\sqrt{\frac{k(T_e + T_i)}{M}} \tag{11}$$

If temperature of ions is small in comparison with electronic one, then

$$j = \frac{en_0}{2}\sqrt{\frac{kT_e}{M}}\,. \tag{12}$$

It is interesting to note, that despite of unsufficient validity of hydrodynamics in collisionless mode, the obtained relation practically coincides with expression

$$j = 0{,}344en_0\sqrt{\frac{2kT_e}{M}} \approx 0{,}49en_0\sqrt{\frac{kT_e}{M}}\,, \tag{13}$$

received at the use of kinetik analysis [2]. As in collision mode the applicability of the hydrodynamical equations looks more proved, the marked concurrence can be used for reception of universal relations for ion emission current density, suitable both in collision and collisionless modes.

3. Collision mode

Let's write down the equation of motion with collision term

$$Mv\frac{dv}{dx} = -e\frac{d\varphi}{dx} - \frac{kT_i}{n}\frac{dn}{dx} - \left(\frac{G}{n} + \nu\right)Mv \tag{14}$$

where ν is effective frequency of ion - atom collisions. Let's consider at first case $G=const$. From the equation of continuity we have $nv = Gx$. Substituting in the previous equation and integrating one receive

$$n(v^2 + v_s^2) - n_0v_s^2 = -\frac{\nu Gx^2}{2} \tag{15}$$

Replacing v on Gx/n we shall receive the equation connecting x and n. At the decision of this equation it is convenient to consider concentration as independent variable. Then the decision can be written down in more compact form

$$x = \frac{v_s}{G}\sqrt{\frac{n(n_0 - n)}{1 + \frac{\nu n}{2G}}} \tag{16}$$

The condition of infringement of quasineutrality $dn/dx \to \infty$ corresponds to condition $dx/dn=0$. Differentiating and equating derivative to 0 one shall find, that quasineutrality is broken

$$n = \frac{n_0}{1 + \sqrt{1 + \frac{\nu n_0}{2G}}} \tag{17}$$

Substituting in expression for x we shall find, that the infringement quasineutrality occurs in a point

$$x_p = \frac{n_0 v_s}{G\left(1 + \sqrt{1 + \frac{\nu n_0}{2G}}\right)}\,, \tag{18}$$

and, accordingly, this expression defines the length of presheath. Let's note, that the length should be much less, than diameter of an electrode, on which the ions are selected. Otherwise task can not be considered one-dimensional. Current density on output from presheath is determined by expression $j=eGx_p$, and thus for ion emission current density we receive

$$j = \frac{en_0\sqrt{k(T_e + T_i)/M}}{\left(1 + \sqrt{1 + \frac{\nu n_0}{2G}}\right)}\,, \tag{19}$$

the speed of ions on presheath – sheath border being equal to $\sqrt{k(T_e + T_i)/M}$, as well as in collisionless mode.

Now we shall consider the case $G=\nu_i n$. In this case one can obtain the following connection between speed and concentration

$$\ln\left(\frac{n_0}{n}\right) = \frac{2\nu_i + \nu}{2(\nu_i + \nu)}\ln\left(\frac{v_s^2 + (1 + \frac{\nu}{\nu_i})v^2}{v_s^2}\right). \tag{20}$$

Analyzing the equation of motion and using the received connection of concentration and speed it is possible to show, that the condition of infringement of quasineutrality is carried out, as well as in the previous cases, at $\nu=\nu_s$. Then we receive for concentration of plasma on border

$$n_b = n_0 \left(2 + \frac{\nu}{\nu_i} \right)^{-\frac{2\nu_i + \nu}{2(\nu_i + \nu)}}, \qquad (21)$$

and for ion emission current density

$$j = e n_0 \sqrt{\frac{k(T_e + T_i)}{M}} \left(2 + \frac{\nu}{\nu_i} \right)^{-\frac{2\nu_i + \nu}{2(\nu_i + \nu)}}. \qquad (22)$$

One can see, that at low pressure, when frequency of ion - atom collisions aspires to 0, the received relation, fair in a case $G=\nu_i n_e$, as well as expression (11), received in the case $G\sim const$, pass in the formula (17), received in collisionless mode.

References

[1] M. D. Gabovich, *Physics and engineering of plasma ion sources,* M.: Mir, 1972.

[2] E. R. Harrison, W. B. Thompson, in Proc. Phys. Soc. 74, 145, 1959.