Comparative Analysis of Calculation Methods of Dense Plasma Radiation

V.I. Oreshkin

2/3 Akademichesky Ave., Tomsk, 634055, Russia, Phone (3822) 492-988, Fax (3822) 491-677, oreshkin@ovpe.hcei.tsc.ru

Abstract – Calculations of the plasma dynamics for a plasma temperature in excess of several electronvolts requires due consideration of the self-radiation of the plasma. In this work, two methods of calculating the radiation of dense plasmas are compared, of which one assumes a local thermodynamic equilibrium and the other uses a more complete nonequilibrium collision-radiation model.

The main difficulties in modelling the behavior of dense high-temperature plasma reside primarily in simulating its self-radiation. As the temperature is elevated, the radiation losses of the plasma grow in importance and, moreover, the local thermodynamic equilibrium appears to be strongly upset. Therefore, the radiation of high-temperature plasmas, e.g., Z-pinch plasmas [1,2] needs to be calculated with the use of nonequilibrium collision-radiation models, taking into account the spectral radiation transport. At the same time, for low-temperature plasma produced, e.g., on electrical explosion of single microconductors [3,4] or early in implosion of wire arrays [5], one can employ simpler radiation models which are based on the approximation of local hydrodynamic equilibrium.

In the latter case, the radiation losses of dense plasma can be taken into account by solving both the angle- and frequency-averaged radiation transfer equations. In this case, transport equations involves only one transport factor – the Rosseland mean free path of a photon L_R: [6]

$$\frac{1}{r}\frac{\partial r W_R}{\partial r} = \frac{1}{L_R}\left(4\sigma_{SB}T^4 - cU_R\right); \qquad (1)$$

$$\frac{1}{3}\frac{\partial U_R}{\partial r} = -\frac{W_R}{cL_R}, \qquad (2)$$

where T is the temperature of matter, W_R is the spectrum-integral radiation flux, U_R is the spectrum-integral radiation energy density, σ_{SB} is Stephan-Boltzmann's constant, c is the velocity of light in vacuum.

Calculation of the Rosseland paths requires determination of the ion charge state and excited level distribution. These parameters were found by solving equations of the ionization equilibrium model (in the local thermodynamic equilibrium approximation, LTE approximation), i.e., by solving the system of Saha-Boltzmann's equations:

$$n_i^k = 1.65\cdot 10^{-22}\frac{g_i^k}{g_1^{k+1}} n_e n_{k+1} T^{-3/2}\exp\left(\frac{I_i^k}{T}\right),$$

where n_i^k is the population of the i-th level of an ion of the k-th ionization factor (k is the spectroscopic index of the ion, T [eV] , I_i^k [eV] is the ionization potential of the i-th level (i is the main quantum number) of an ion of the k-th ionization factor, g_i^k is the statistical weight of the i-th level of an ion of the k-the ionization factor, n_e and n_i^k [cm^{-3}].

In calculating the absorption coefficients, account was taken of free-free electron transitions, free-coherent transitions, and also absorption in spectral lines.

In the continuous spectrum, absorption of photons occurs by two mechanisms: deceleration and recombination. The deceleration absorption coefficient is determined by the expression:

$$k_\nu^{Br} = \left(1-\exp\left\{-\frac{h\nu}{kT}\right\}\right)\sum_k n^k \sigma_k^{Br}(\nu, T),$$

where $n^k = \sum_i n_i^k$ is the density of ions of the k-th ionization factor, σ_k^B cross-section for deceleration absorption:

$$\sigma_k^{Br}(\nu, T_e) = \frac{2.43\cdot 10^{-37}(k+1)^2 n_e}{(h\nu)^3\sqrt{T}} \quad [\text{cm}^2],$$

$h\nu$ and T in [eV].

The recombination absorption coefficient is equal to:

$$k_\nu^{\mathrm{Re}k} = \left(1-\exp\left\{-\frac{h\nu}{kT}\right\}\right)\sum_{i,k} n_i^k \sigma_{ik}^{\mathrm{Re}k}(\nu),$$

where $\sigma_{ik}^{\mathrm{Re}k}$ is the cross-section for recombination absorption which is determined by the expression:

$$\sigma_{ik}^{\mathrm{Re}k} = 7.91\cdot 10^{-18}\frac{i}{(k+1)^2}\left(\frac{I_i^k}{h\nu}\right)^3 [\text{cm}^2],$$

at $h\nu \ge I_i^k$.

The photon absorption coefficient in a spectral line with account of the forced radiation is determined by the expression:

$$k_\nu^l = \frac{h\nu}{4\pi}\left(B_{ij}^k n_j^k - B_{ji}^k n_i^k\right)\psi_\nu ,$$

where h is Planck's constant, ψ_ν is the line profile (in calculations, convolution of Doppler and natural broadening profiles was employed), B_{ij}^k, B_{ji}^k are the Einstein coefficients for the probability of forced radiation and absorption which are related to each other and to the Einstein coefficient for the probability of spontaneous radiation A_{ij}^k by the following relations:

$$A_{ij}^k = \frac{2h\nu^3}{c^2} B_{ij}^k ; \quad g_j^k B_{ij}^k = g_i^k B_{ji}^k .$$

The foregoing procedure was realized in the RACE-I program which allows calculating the equilibrium ionization distribution of the plasma, the spectral dependence of the absorption factor, and different averages from this dependence (in the form of tables in which the mean free path lengths depend parametrically on the plasma density and temperature) for a matter of atomic number from 1 (hydrogen) to 36 (krypton). Moreover, the RACE-I program provides a possibility of calculating these parameters for plasma composed of a mixture of different matters.

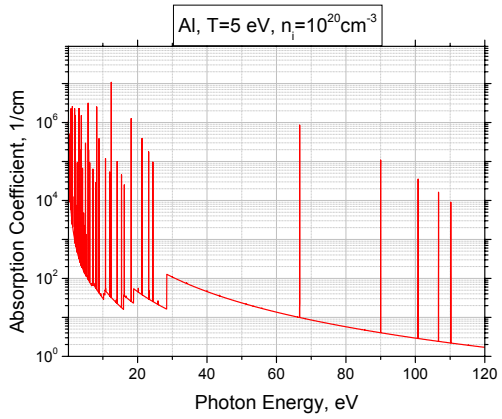

Fig.1 Spectral dependence of the absorption coefficient for aluminum.

Examples of calculations of the spectral dependence of absorption coefficients for aluminum and copper are given in Figs.1 and 2. Once the spectral dependence of the absorption coefficients k_ν, was determined, the Rosseland mean free paths of photons were found according to the expression [6]:

$$L_R = \frac{\int_0^\infty \frac{1}{k_\nu}\frac{dU_\nu^{BB}}{dT}d\nu}{\int_0^\infty \frac{dU_\nu^{BB}}{dT}d\nu} , \qquad (3)$$

where $U_\nu^{BB} = \frac{8\pi h\nu^3}{c^3}\left(\exp(\frac{h\nu}{kT})-1\right)^{-1}$ is the density of the "black body" energy at a frequency ν.

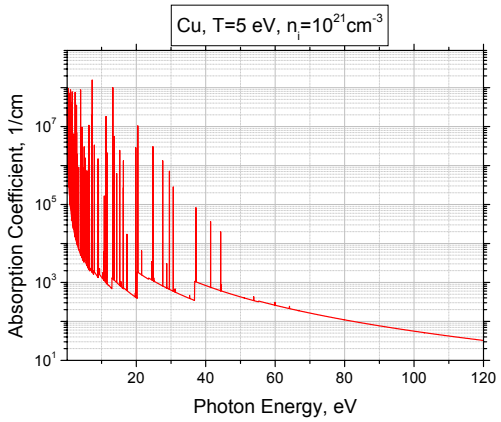

Fig.2 Spectral dependence of the absorption coefficient for copper.

Tables of the Rosseland mean-free paths of photons L_R have been drawn for aluminum and copper. These tables allow calculations of the plasma radiation which is what we did for homogeneous Al and Co plasma columns within the framework of the diffusion approximation (Diffusion Approach, DA) or in the "grey" body approximation [6]. The results of calculations are presented in Figs.3 and 4 where the temperature dependences of the radiation power losses of a homogeneous plasma column and also the dependences of the Rosseland mean-free paths of photons are shown.

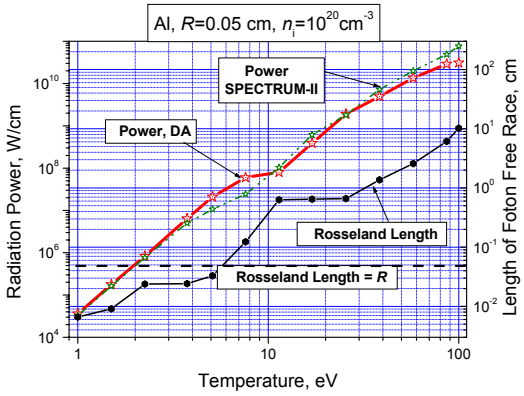

Fig.3 Temperature dependence of the radiation losses of a homogenous Al column calculated by different methods.

The results of calculations of the plasma column radiation were compared with calculations by the SPECTRUM-II program [7] in which the charge state and excited level distributions were found using a nonequilibrium nonstationary shock-radiation model. Radiation reabsorption in spectral lines was taken into account with the use of eluding factors and the radiation field of the plasma column was determined by solving the transport equations for different directions and different frequencies. The results of calcu-

lations of the radiation losses of a homogenous plasma column (frequency-integrated radiations flux) are shown in Figs.3 and 4 by dash-dot lines.

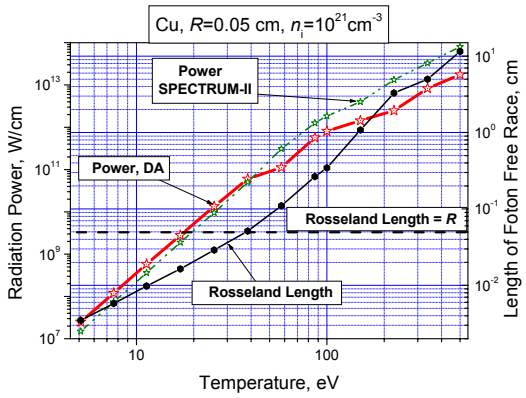

Fig.4 Temperature dependence of the radiation losses of a homogeneous Co column calculated by different methods.

Comparison of the calculations of the radiation losses by the SPECTRUM-II program with those made in the "grey" body approximation shows that in the range of applicability of the "grey" body approximation ($L_R << R_w$, where R_w is the characteristic size of the emitting region, in our case the radius of the plasma column) the both methods ensure nearly identical results. However, a good agreement obtains even with Rosseland paths equal to the characteristics size of the emitting region (horizontal dashed line in Figs.3 and 4), and notable discrepancies are found only when L_R is two orders of magnitude larger than R_w. Thus, the "grey" body approximation can be applied not only in the range of its formal applicability, but up to $L_R \approx R_w$.

The range of applicability of the LTE-methods of calculations of the plasma radiation can be expanded using a multigroup approximation. In this case, the spectral range is divided into groups differing in photon energy. For the k-th group, the lower limit (the lowest photon energy) is E_k and the upper limit is E_{k+1}. The equations of radiation transport (1,2) are applicable in the multigroup approximation, but require to be changed somewhat. First, for each group there must be its own table of Rosseland paths calculated in accord with expression (3), in which the limits of integration are changed to E_k and E_{k+1}. Second (for the changed mean free path), transport equations (1, 2) must be solved using a different function of sources:

$$4\sigma_{SB}T^4 \rightarrow B(x_k, x_{k+1})4\sigma_{SB}T^4,$$

where $x_k = \frac{E_k}{kT}$. The factor $B(x_k,x_{k+1})$ is calculated in accordance with the expression:

$$c\int_{E_k}^{E_{k+1}} U_\nu^{BB} d\nu = B(x_k, x_{k+1})4\sigma_{SB}T^4.$$

For the factor $B(x_k,x_{k+1})$, we can write the expressione:

$$B(x_k, x_{k+1}) = B(x_{k+1}) - B(x_k),$$

wheree $B(x) = \frac{15}{\pi^4}\int_0^x \frac{x^3}{\exp(x)-1}dx$. For the factors B(x), there exist the following approximate expressions [8]:

при $x \le 2$

$$B(x) = \frac{15}{\pi^4}x^3\left(\frac{1}{3} - \frac{x}{8} + \frac{x^2}{62.4}\right);$$

at $x > 2$

$$B(x) = 1 - \frac{15}{\pi^4}\exp(-x)\left(x^3 + 3x^2 + 6x + 7.2688\right)$$

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