

# Dynamic Current Multiplier<sup>1</sup>

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**Abstract – The Load Current Multiplier (LCM) concept was proposed to increase the energy transfer efficiency into pulse-power loads [1]. Using the analytical approach suggested in [1], we define a dynamic current multiplier configuration (DCM) allowing also power increase in the circuit. The parameters of multiplier dynamically vary in time in this concept. This allows current pulse sharpening through rapid transfer of the stored magnetic flux to load. DCM is shown to provide theoretically higher load power than in other considered schemes and it is potentially applicable at existing and future multi-MA generators.**

## 1. Introduction

Modern high energy density physics applications require energy release times essentially less than one microsecond. On the other hand, pulse-power capacitors allow easier generation of MA currents,  $I_0$ , with microsecond rise-times,  $t_0$  [2, 3]. The minimum  $t_0$  and maximum  $I_0$  values are usually restricted by irreducible inductances in the generator circuit. As an example, consider simple discharge of a capacitor,  $C$ , through the inductance,  $L_0$ , and through an inductive load  $L_d$ . If resistive losses are neglected,  $I_0$  and  $t_0$  are

$$I_0 = U_0 \sqrt{C/(L_0 + L_d)}, \quad t_0 = \pi \sqrt{C(L_0 + L_d)}/2 \quad (1)$$

where  $U_0$  is the initial capacitor voltage. The minimum  $L_0$  value is limited in practice by the electrical or magnetic insulation strength. Eq. (1) thus typically results in  $t_0 \geq 1 \mu\text{s}$  for direct load-to-generator connection [2, 3] (direct drive). Sharpening of the current pulse requires partial conversion of the maximum stored magnetic energy in  $L_0$  into electric energy. The energy transformation supporting the power flux to the load can be controlled by a variable resistor or inductor connected in parallel to  $L_d$ . This electric-to-magnetic-to-electromagnetic conversion represents essential part of power conditioning in IES systems.

## 2. Power conditioning

For example, consider a plasma armature motion corresponding to increasing inductance  $L_u(t)$  of an intermediate storage volume loaded by generator [4]. The capacitor is first discharged through inductance ( $L_0 + L_u$ ) and  $L_d$  is not connected until the current reaches its maximum value  $I_g$  at  $t = t_g$ . Let the inductance in-

stantaneously change at this moment from  $L_u$  to  $M$  and connect the load in parallel to  $L_u$ . The charge and magnetic flux conservations imply the change of currents [4] (resistive losses will further be always neglected),

$$\begin{aligned} (L_0 + L_u) I_g &= L_0 J_g + M J_u \\ L_u I_g &= M J_u - L_d J_d, J_g = J_u + J_d \end{aligned} \quad (2)$$

where  $J_g$ ,  $J_u$ , and  $J_d$  are new generator, inductive volume and load current values accordingly. We normalize inductances to the generator inductance value introducing  $u \equiv L_u/L_0$ ,  $x \equiv M/L_0$ ,  $d \equiv L_d/L_0$  and we define the load current multiplication coefficient,  $\kappa \equiv J_d/I_0$ ,  $I_0$  is taken from Eq. (1). Eq. (2) yields

$$\begin{aligned} \kappa \equiv |J_d/I_0| &= \frac{x-u}{x+d+xd} \sqrt{(1+d)/(1+u)} \\ \tau \equiv t_g/t_0 &= \sqrt{(1+u)/(1+d)} \end{aligned} \quad (3)$$

where  $\tau$  is the capacitor discharge time normalized to  $t_0$  from Eq. (1). In its turn, the load current rise-time is fully defined by the time,  $\Delta t$ , of  $L_u$ -to- $M$  variation assumed to be faster than the capacitor discharge. The  $d(L_u I_u)/dt$  term creates the energy flux and load power multiplication with respect to direct drive.

The energy transfer coefficient can be defined as  $\eta \equiv L_d J_d^2 / C U_0^2 = k^2 d / (1+d)$ . In the above, we define the load as a small volume with high magnetic energy density and low inductance  $L_d$ . At least during a portion of the current rise-time, the inequality  $L_d \ll L_0$  ( $d \ll 1$ ) is satisfied in many examples from pulse-power physics [1]. As a result, the load magnetic energy is a small fraction of the total available magnetic energy in both cases, (1) and (3).

## 3. LCM

Reference [1] introduces the concept of the Load Current Multiplier (LCM) capable to improve the generator-to-load energy coupling in direct-drive configurations with  $d \ll 1$ . The scheme suggested in [1] is shown in Fig. 1 in its simplest configuration,  $N = 2$ . Assume the load inductance  $L_d$  and the coupled and uncoupled inductances,  $L_c$  and  $L_u$  in Fig. 1 are constant and the switch  $S$  is closed. The magnetic flux  $\Psi$  is provided by a capacitor  $C$ . According to the analysis of [1], the charge and magnetic flux conservations imply

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$$\begin{aligned}\Psi &= L_0 I_g + L_u I_g + L_c I_c + L_d I_d \\ L_c I_c - L_d I_d &= 0 \\ I_c &= 2I_g - I_d\end{aligned}\quad (4)$$

Or, introducing normalized inductances as above with  $c \equiv L_c/L_0$ , and considering the coefficient  $\kappa = I_d/I_0$  at maximum load current, we get from (4):

$$\begin{aligned}\kappa &= \frac{2c}{c+d} \sqrt{\frac{1+d}{1+u+4cd/(c+d)}} \\ \tau &= 2c/\kappa(c+d)\end{aligned}\quad (5)$$

An ideal LCM ( $u \ll 1 \ll c$ ) would provide better energy transfer to load without increasing the current rise-time very much if compared to direct-drive connection when  $d \ll 1$ . This scheme property was experimentally confirmed for constant-inductance loads [1]. However, while increasing indirectly the load power through current increase, multiplier does not shorten the load current rise-time.

#### 4. Dynamic current multiplier

Now assume that the values characterizing the multiplier in the *rhs* of Eq. (4),  $L_u$  and  $L_c$ , one of them or both may vary in time. Indeed, the tori in Fig. 1 connected in parallel to the load represent an intermediate energy and magnetic flux storage. Dynamic change of these values, or introduction of varying resistances in the scheme would possibly lead to the flux ousting to the secondary with sharpening of the load current pulse. Therefore, in this paper we try to respond the question: How variation of multiplier parameters will change magnetic flux distribution between the storage and the load volumes? In particular, the consideration might be similar to that of system (2). Below we introduce three possible configurations of a Dynamic Current Multiplier (DCM).

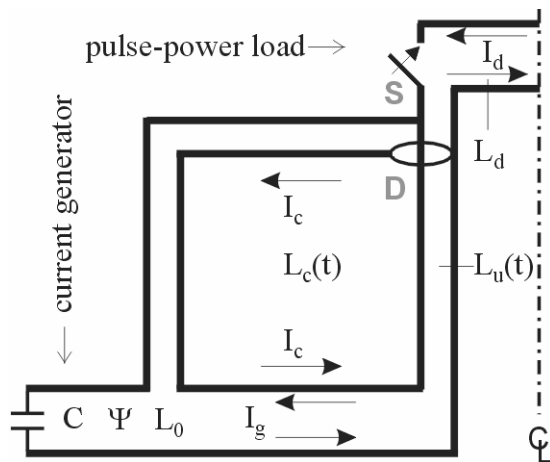


Fig. 1. Cylindrical LCM from [1],  $N = 2$ . Concentric tori are connected to generator through convolute D. S is a closing switch connecting load to the tori. Generator current  $I_g$  splits into surface currents  $I_c, I_d$ .

#### 4.1. DCM1

The switch S in Fig. 1 is open until the capacitor discharge current reaches its maximum value  $I_g$  at  $t = t_g$ . The flux supplied by C and the discharge time are

$$\begin{aligned}\Psi &= L_0 I_g + L_u I_g + 2L_c I_c \\ I_c &= 2I_g, \tau = \sqrt{(1+u+4c)/(1+d)}\end{aligned}\quad (6)$$

where  $\tau$  is defined as in (3) and (5).

Further, the load is connected and the inductance  $L_u$  changes to the value M, both events occur instantaneously and at the same time  $t = t_g$ . Similarly to Eq. (2), the magnetic flux is conserved at the new current values J are

$$\begin{aligned}(L_0 + L_u + 4L_c)J_g &= L_0 J_g + M J_g + 2L_c J_c \\ L_c I_c &= L_c J_c - L_d J_d, J_c + J_d = 2J_g\end{aligned}\quad (7)$$

This describes current interruption in the primary and its transfer to the connected secondary. The system was studied in [5] for the case of dielectric-insulated transformers (a solid fuse used for flux redistribution). For  $k \equiv |J_d/I_0|$  we have

$$\kappa = \frac{2c(x-u)}{c(1+x+4d)+(1+x)d} \sqrt{\frac{1+d}{1+u+4c}}\quad (8)$$

At a fixed  $x$ ,  $\kappa \rightarrow 0$  both for  $c \rightarrow 0$  and  $c \rightarrow \infty$ . The maximum  $\kappa$  corresponds to some optimum coupled inductance value,  $c_{opt}(x)$ , given by the expression

$$c_{opt} = \alpha \left( 1 + \sqrt{1 + \frac{1+u}{\alpha}} \right), \alpha \equiv \frac{(1+x)d}{2(1+x+4d)}\quad (9)$$

The load current can be further increased if we short-circuit the generator,  $L_0$ , simultaneously with the above-considered events (“ $1+x$ ” should be simply replaced by “ $x$ ” in Eq. (8)). In this case, we have

$$\begin{aligned}\kappa &= \frac{2c(x-u)}{c(x+4d)+xd} \sqrt{\frac{1+d}{1+u+4c}} \\ \alpha &= xd/2(x+4d)\end{aligned}\quad (10)$$

For the vacuum multiplier configuration of [1], this improvement was considered in [6]. We note a higher magnetic flux transfer in Eq. (10) than in (8), but at a more strict requirements on the power multiplication element,  $x \gg 4d$  in Eqs. (8) and (10). Fig. 2 compares the inductive storage times and the current transfer coefficients for the described configuration, DCM1, and for those of Eqs. (3) and (5) ( $\kappa = \tau \equiv 1$  for direct drive). The energy transfer efficiencies can always be found as  $\eta = k^2 d / (1+d)$ . It can be seen that at a reasonable value of  $x$ ,  $x = 1$ , the DCM1(b) scheme ( $L_0$  is cut from the rest of the circuit) corresponds to  $\kappa < 1$ , but to higher  $\kappa$  values than DCM1(a), and to the same  $k$  as in the inductive scheme of Eq. (2). Substantial advantage with respect to Eq. (2) is achievable only for larger  $x$  or/and smaller  $d$  and  $u$  than those considered.

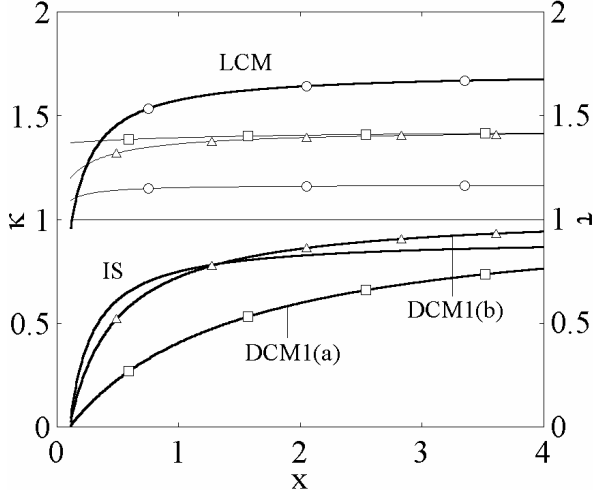


Fig. 2. Comparison of  $\kappa$  (thick lines) and  $\tau$  (thin lines) for inductive switch (IS), LCM and DCM1 ((a) from Eq. (8) and (b) from Eq. (10)) for  $d = u = 1/10$ :  $c = x$  for LCM, Eq. (5), and  $c = c_{opt}(x)$  for DCM1.

#### 4.2. DCM2

We continue our study by considering that the coupled inductance of the multiplier in Fig. 1 is varying in time from a small value  $L_c$  to a large value  $M$ . The capacitor discharge and the moment of maximum generator current  $t_g$  are still defined by Eq. (6). Further fast  $L_c$  change and corresponding flux redistribution are described by system (7) with the first equation replaced by that of Eq. (11) and the current multiplication becomes

$$L_0 I_g + L_u I_g + 2L_c I_c = L_0 J_g + L_u J_g + 2M J_c$$

$$\kappa = \frac{2(1+u)(x-c)}{(1+u)(x+d) + 4xd} \sqrt{\frac{1+d}{1+u+4c}} \quad (11)$$

where the normalized inductances  $d$ ,  $c$ ,  $u$ , and  $x$  are defined as for Eqs. (3, 5). If compared to the result of Eqs. (8, 10), the requirements on the power multiplication element are relaxed here ( $x \gg d$  is necessary for ideal operation, instead of  $x \gg 4d$ ). Coefficient  $\kappa$  has a maximum at some  $u = u_{opt}(x)$

$$u_{opt} = \alpha \left( 1 + \sqrt{1 + 16c/\alpha} \right) - 1, \alpha \equiv 2xd/(x+d) \quad (12)$$

which must be positive.

As the inductances  $L_u$  and  $L_c$  are small in the considered configuration, the energy in intermediate storage is also small and the primary storage inductance  $L_0$  should be kept in the circuit during power multiplication and energy transfer to the load.

#### 4.3. DCM3

Consider now that both  $L_c$  and  $L_u$  are varying in time and they obey  $L_c(t) + L_u(t) = \text{const}$ . Thus, at  $t = t_g$ , the coupled inductance in Fig. 1 instantaneously changes

from  $L_c$  to  $L_c + M$  and the uncoupled one from  $L_u + M$  to  $L_u$ . The capacitor discharge will be described by

$$\Psi = L_0 I_g + (L_u + M) I_g + 2L_c I_c$$

$$I_c = 2I_g, \tau = \sqrt{(1+u+x+4c)/(1+d)} \quad (13)$$

Again, only the first equation of (7) is modified when considering magnetic flux conservation after  $L_d$  connection and  $L_u$  and  $L_d$  change. Thus, we have:

$$L_0 I_g + (L_u + M) I_g + 2L_c I_c =$$

$$= L_0 J_g + L_u J_g + 2(L_c + M) J_c \quad (14)$$

$$\kappa = \frac{2x(1+u+c+x)}{(1+u)(c+x+d) + 4(c+x)d} \frac{1}{\tau}$$

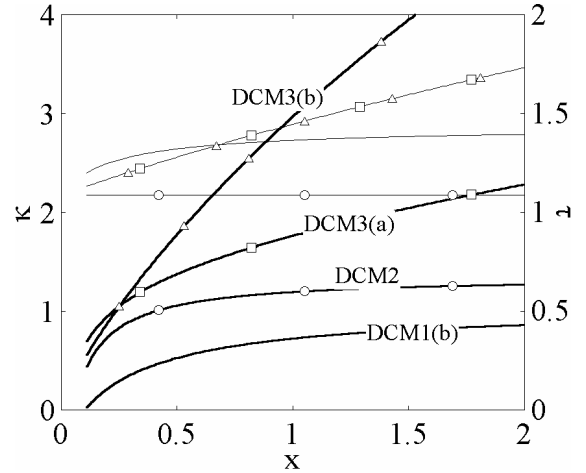


Fig. 3. Load current multiplication  $\kappa$  (thick lines) allowed by magnetic flux conservation and  $\tau$  values (thin lines) for  $d = 1/10$ : DCM1(b) (Fig. 2,  $u = 1/10$ ,  $c = c_{opt}$ ), DCM2 (Eq. (11),  $c = 1/20$ ,  $u = u_{opt}$ ) and DCM3 ((a) from Eq. (14) and (b) from Eq. (15),  $u = 1/10$ ,  $c = 1/20$ ).

Now the intermediate stored energy is substantial and for  $L_0$  cut-off, in analogy with Eq. (10) we thus have

$$\kappa = \frac{2x(u+c+x)}{u(c+x+d) + 4(c+x)d} \frac{1}{\tau} \quad (15)$$

Analytical efficiencies for the described DCM schemes are compared in Fig. 3. If the magnetic energy storage time  $\tau$  is not constrained, the best operation is demonstrated by the DCM3 configuration ( $\tau = 1.45$  at  $x = 1$  and (a)  $\kappa = 1.76$ , (b)  $\kappa = 2.97$ ). Surprisingly, Eqs. (14, 15) formally yield  $\kappa \propto x^{1/2}$  at  $x \rightarrow \infty$ . In practice, however, if no external energy is supplied the load current is limited by the energy conservation in the system. We postpone this discussion to elsewhere and note only that for the parameters of Fig. 3 and for the conservative system of Figs. 4, 5 below, the DCM3 design corresponds to (a)  $\kappa_{max} \approx 1.32$ , (b)  $\kappa_{max} \approx 1.53$ .

5. Example of applications

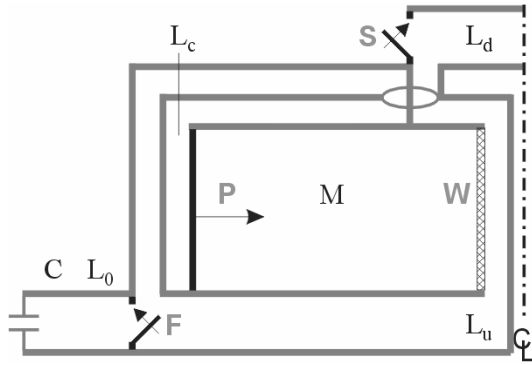


Fig. 4. Example of DCM geometry with closing switches, S, F, and a hollow cylindrical conductor providing  $L_u$  and  $L_c$  variation. The armature P is magnetically accelerated towards the wall W (conducting for DCM2 or dielectric for DCM3).

Fig. 4 presents an example of concrete experimental arrangement for DCM2 and DCM3 configurations. The corresponding circuit equations are further derived from (11) and (14) and completed by the equation of motion for a perfectly conducting, infinitively thin, microsecond-compression [2, 3] cylindrical conductor P in Fig. 4. We fix the circuit parameters to  $C = 7.4 \mu\text{F}$ ,  $U_0 = 0.6 \text{ MV}$ ,  $L_0 = 50 \text{ nH}$  ( $t_0 = 1 \mu\text{s}$  and  $I_0 \approx 7 \text{ MA}$ ), and  $L_d = L_u = 5 \text{ nH}$ ,  $L_c = 2.5 \text{ nH}$ .

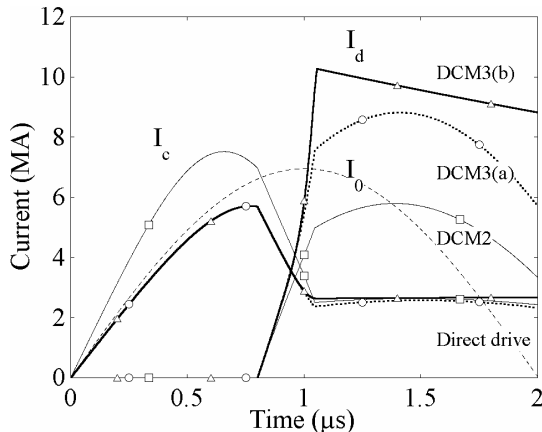


Fig. 5. Numerical result for DCM configuration of Fig. 4. The currents of multiplier,  $I_c$ , and load,  $I_d$ , are compared to direct-drive current  $I_0$  (d, u, and c are the same as in Fig. 3).

The conductor P has the height and initial radius both equal to 10 cm and a ten-fold compression is

assumed. The shell mass was chosen to have implosion at  $t = 1 \mu\text{s}$  and it was equal to 19 mg for DCM2 and to 6 mg for DCM3. The load was connected at  $t = 0.8 \mu\text{s}$  in all cases,  $L_0$  cut-off occurred at  $t = 0.9 \text{ ns}$  for DCM3(b). The load current rise-time was constrained by 250 ns.

Fig. 5 shows numerical solutions for the design of Fig. 4 and illustrates substantial load power multiplication with load current amplification if compared to direct drive. Practical limitation on  $\kappa$  is defined by partial magnetic-to-kinetic energy transfer in this configuration and by possible conductor bouncing when the accelerating magnetic force changes direction.

4. Conclusion

In conclusion, we described a possible approach to vacuum power multiplication in IES systems. The concept assumes dynamic variation of proper characteristics of the vacuum current multiplier suggested in [1]. Analytical and numerical results for the introduced Dynamic Current Multiplier schemes show benefits of its usage with respect to direct-drive, or to conventional schemes with opening switches. DCM for  $N = 2$  is considered, but the scheme properties stand for  $N > 2$  too. We note also that our main conclusions remain valid for the design of Fig. 1 if flux redistribution is organized in a different way than that of Figs. (4, 5). For example, a ferromagnetic opening/closing switch may be used instead of moving conductor, or a resistive opening switch having resistance  $R \sim M/\Delta t$  would also ensure good DCM operation. Externally controlled inductive variations with additional energy supply in the scheme would provide, according to Fig. 3, further increase of the load power.

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