Reflection of the Brillouin Flow from the Load in MITL

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Abstract – In the report, an analytical theory is presented describing the reflection of the Brillouin flow [1-2] from the load in a magnetically insulated transmission line (MITL). It supports the conclusion made previously [3] that the reflected wave efficiently converts the vacuum electrons to cathode current, and propagates back to the generator with a velocity depending on the mismatching between the load and the impedance of the forward going wave. The predictions of the theory are in good agreement with PIC simulations presented in [3], and support the conclusion [3] that at high enough voltage the reflected wave "sees" the vacuum impedance of the MITL.

1. Introduction

The Brillouin (or parapotential) flow in a MITL is a flow where the electrons behind the front edge of the wave are moving along the equipotential surfaces. The theory [1-2] predicts that the Brillouin flow may have infinite number of the flow patterns that differ by the potential of the most distant electrons, γ_m. The total current (in kA) in the Brillouin flow is given by

$$I(\gamma_m) = \frac{511}{z_V}\gamma_m [ln(\gamma_m + \sqrt{\gamma_m^2 - 1}) + \frac{\gamma_a - \gamma_m}{\sqrt{\gamma_m^2 - 1}}] , \quad (1)$$

where z_V is the MITL vacuum impedance in Ohms, $\gamma_a = 1 + U_a/511$, U_a is the anode voltage in kV. The function $I(\gamma_m)$ has a minimum $I_{min}(\gamma_m{}^*)$, where $\gamma_m{}^*$ is the solution of the equation

$$\frac{dI(\gamma_m)}{d\gamma_m} = 0 . \quad (2)$$

When the driver with high enough output voltage is connected to the entrance of the MITL, and the cathode begins to emit the electrons, the leakage current on the front edge of the wave appears allowing the wave to propagate downstream. Theory claims that the wave can't propagate in MITL if the leakage current is zero, because the displacement current only does not generate enough magnetic field to isolate the flow behind the front edge of the wave. If the driver is not able to provide enough leakage current, the wave can't propagate, all the electrons cross the AK gap at the entrance of the line. As soon as the sum of the leakage current and displacement current reaches the $I_{min}(\gamma_m{}^*)$, the wave may propagate downstream. Below the model which assumes that the forward going wave carries the minimum current, $I_{min}(\gamma_m{}^*)$, is referred as a I_{min} model.

Let us consider now what may happen if the driver is able to provide at a same output voltage more current than I_{min}. At some $I_{max} > I_{min}$ the magnetic field becomes so strong that the height of the electron cycloid on the front edge of the wave becomes equal to the AK gap in MITL. Further increase of the current is not possible because the electrons will not be able to cross the AK gap, and the leakage current would reduce to zero. So, we have to conclude that in real MITL may propagate only the waves with the current I_a, which satisfies the condition: $I_{min} < I_a < I_{max}$.

The maximum possible MITL current I_{max} can be defined [4] as a current that generates a magnetic field which curves the trajectory of the electrons crossing the AK gap in such a way that the angle φ between the trajectory and the anode surface equals to zero, $\varphi = 0$. As soon as this angel is specified, the Eq. (1) predicts only one possible solution $I_{max}(\gamma_{mx})$ with definite $\gamma_m = \gamma_{mx}$, which is determined by the equations [4]:

$$\begin{cases} 2\gamma_{mx}(\gamma_a - 1) - 2(A-1)\dfrac{(\gamma_a + \gamma_{mx}^2 - \gamma_{mx} - 1)}{(\gamma_a\gamma_{mx} - 1)} = \\ = \{2 + \sqrt{\gamma_{mx}^2 - 1}\cdot ln(\gamma_{mx} + \sqrt{\gamma_{mx}^2 - 1}) + \\ + 2\gamma_{mx}^2(\gamma_a - 1) - \gamma_a - \gamma_{mx}\}\dfrac{\gamma_a - 1}{(\gamma_a\gamma_{mx} - 1)}, \\ A = \dfrac{(\gamma_a\gamma_{mx} - 1)^2 + (\gamma_{mx}^2 - 1)(\gamma_a - 1)^2}{(\gamma_a\gamma_{mx} - 1)^2 - (\gamma_{mx}^2 - 1)(\gamma_a - 1)^2}. \end{cases} \quad (3)$$

At $0 < U_a$, MV < 20, the solution of the Eqs. (3) can be approximated by

$$\gamma_{mx} \sim 1 + 0.3086U_a - 0.00951U_a^2 + 2.213\cdot 10^{-4}U_a^3 .$$

Figure 1 shows the curves $\gamma_{mx} = f(U_a)$ and $\gamma_m{}^* = f(U_a)$. They intersect at $U_a \sim 3$ MV, at higher voltage γ_{mx} exceeds $\gamma_m{}^*$ corresponding to the right branch of the Brillouin curve $I(\gamma_m)$, at lower voltage γ_{mx} is less than $\gamma_m{}^*$, the operating point of the MITL lies on the left branch of the Brillouin curve. It means that at high voltage much less current may flow in the cathode than it can be expected by using the I_{min} model.

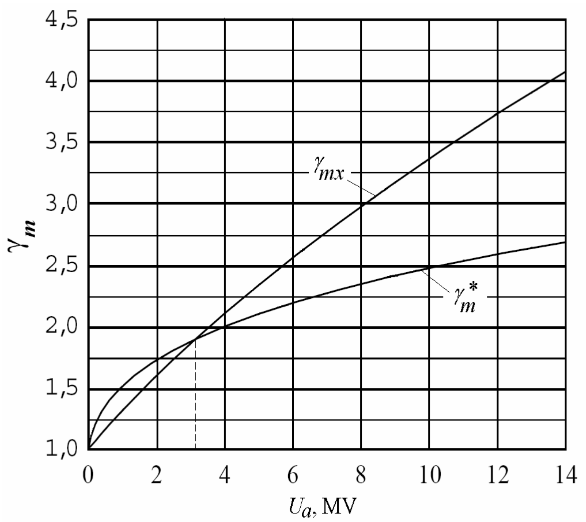

Fig. 1. Potential at the outer boundary of the electron cloud γ_{mx} and γ_m* versus anode voltage in MITL.

The interesting feature of the Brillouin flow is that in spite of the values of γ_{mx} and γ_m* differ sufficiently, especially at high voltage, the currents $I_{max}(\gamma_{mx})$ and $I_{min}(\gamma_m^*)$ at U_a exceeding ~1 MV are rather close (see Fig. 2).

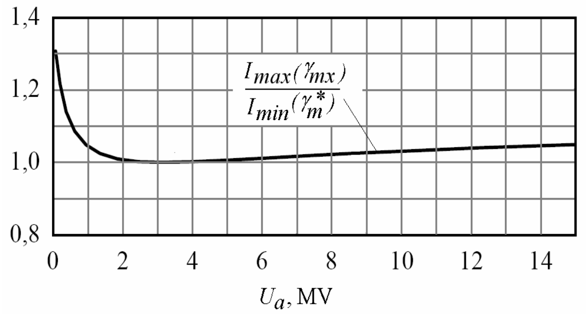

Fig. 2. The ratio between the maximum $I_{max}(\gamma_{mx})$ and the minimum $I_{min}(\gamma_m^*)$ currents versus voltage.

Below the model which assumes that the forward going wave carries the maximum current, $I_{max}(\gamma_{mx})$, is referred as a I_{max} model.

2. Reflection of the Brillouin flow from the load

Let us consider now the reflection of the forward going wave from the load with impedance z_L. Below the parameters related to the forward going wave are denoted by "1", the parameters of the flow behind the front edge of the reflected wave are denoted by "2". All the currents are in kA, the voltages are in kV, and the impedances are in Ohms. The input parameters for the theory are the parameters of the forward going wave, the vacuum impedance of the MITL, and the impedance of the load.

After the reflection, there are three parameters that describe the flow behind the front edge of the reflected wave, U_2, I_2, and γ_{m2}. The fourth useful parameter is the velocity of the reflected wave, V_R. To determine these four parameters four equations are needed.

First is the Eq. (1), that determines now the total current $I_2(U_2, \gamma_{m2})$ in the Brillouin flow behind the front edge of the reflected wave.

Second equation is the Ohm's law: $I_2 z_L = U_2 = 511(\gamma_{a2} - 1)$.

Third equation is the charge conservation law:

$$\begin{cases} I_1 - I_2 = (\rho_2 - \rho_1)V_R, \\ \rho_1 = \frac{I_1}{c}\sqrt{1-\frac{1}{\gamma_{m1}^2}}, \rho_2 = \frac{I_2}{c}\sqrt{1-\frac{1}{\gamma_{m2}^2}} \end{cases} \quad (4)$$

where c is the speed of light.

Fourth equation appears from the standard procedure used to determine the speed of the reflected wave. In the inertial system which is moving with the speed of the wave, the corresponding electric $\bar{E}'$ and magnetic $\bar{H}'$ fields do not change in time, therefore

$$rot\,\bar{E}' = -\frac{1}{c}\frac{\partial \bar{H}'}{\partial t} = 0\,. \quad (5)$$

In this case the Stocks theorem yields

$$\oint \bar{E}'\,d\,\bar{\ell} = 0\,, \quad (6)$$

where the integration is made along the closed contour shown by dashes in Fig. 3. In the laboratory coordinates Eq. (6) transfers into

$$\int (E + \beta_R H)d\ell = 0\,, \quad (7)$$

where $\beta_R = V_R/c$, and the integration is made along the left and right sides (with opposite $d\ell$) of the same contour in Fig. 3.

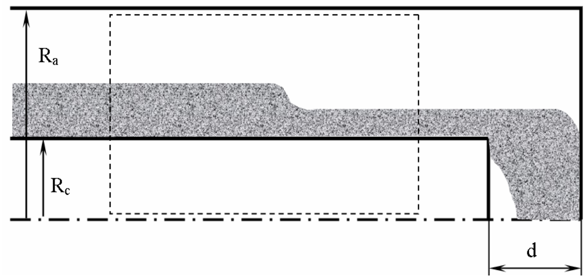

Fig. 3. Part of the MITL with reflected wave showing the integration contour in Eqs. (6-7).

The Eq. (7) can be reduced to the forth equation which is needed to determine the parameters of the flow behind the reflected wave:

$$\gamma_{a1} + \beta_R\sqrt{\gamma_{m1}^2 - 1} + \beta_R\frac{\gamma_{m1}(\gamma_{a1} - \gamma_{m1})}{\sqrt{\gamma_{m1}^2 - 1}} = \\ = \gamma_{a2} + \beta_R\sqrt{\gamma_{m2}^2 - 1} + \beta_R\frac{\gamma_{m2}(\gamma_{a2} - \gamma_{m2})}{\sqrt{\gamma_{m2}^2 - 1}}\,. \quad (8)$$

3. Results

The four equations described above can be solved numerically for any input parameters U_{a1}, I_1, γ_{m1}, z_V, and z_L. They predict that the forward going wave reflects from the load if only $z_L < z_1$, where $z_1 = U_{a1}/I_1$ is the MITL impedance which "sees" the forward going wave. At $z_L = z_1$ the velocity of the reflected wave is zero, it increases if the ratio z_L/z_1 reduces. An example of I_{max} model predictions at U_{a1} = 8.7 MV is shown in Fig. 4. The value of γ_{m2} behind the reflected wave is below ~1.9 at any z_L, whereas in the forward going wave γ_{m1} ($=\gamma_{mx}$) exceeds ~3 (see Fig. 1).

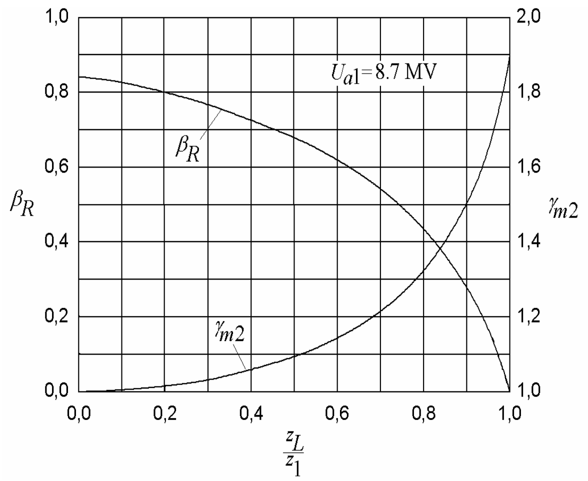

Fig. 4. I_{max} model predictions at U_{a1} = 8.7 MV.

Table 1 gives summary of the PIC simulations presented in [3] in comparison with several models. Here, z_V model assumes that the current in the forward going wave is $I_{min}(\gamma_m*)$, and the reflected wave "sees" the vacuum impedance of the MITL [3]. The velocity of the reflected wave in z_V model is determined by using the Eq. (4).

4. Discussion

The results of the PIC simulation are taken from the plots presented in [3], therefore the accuracy of these data is not as high, except β_R. Nevertheless, the predictions of all the models are very similar, and close to what is observed in PIC simulations. The main difference is the value of γ_{m1} which is ~2.25 in PIC simulation compared to 2.395 or 2.125 predicted by I_{max} and I_{min} models, respectively. Note, that PIC simulation predicts for the forward going wave the point U_{a1} = 5.25 MV, z_V = 55.4 Ohm, I_1 ~125.9 kA, γ_{m1} ~ 2.25 that does not stay on the Brillouin curve (which requires γ_{m1} ~ 2.67 for such I_1). The I_{max} model overestimates, and both the I_{min} and z_V models slightly underestimate the re-trapping effect, that can be characterized by $(I_{C2}/I_2)/(I_{C1}/I_1) = \gamma_{m1}/\gamma_{m2}$. The I_{max} model predicts the best fit for the velocity of the reflected wave.

Table 1. Comparison between the PIC simulations [3] and several analytical models.

Forward going wave (U_{a1} = 5.25 MV, z_V = 55.4 Ohm)		
Model	γ_{m1}	I_1 kA
PIC	~2.25	~125.9
I_{max} model	2.395	123.7
I_{min} model	2.125	122.8
z_V model	2.125	122.8

Flow behind the reflected wave						
Model	U_{a2} MV	I_2 kA	γ_{m2}	γ_{m1}/γ_{m2}	β_R	z_L Ohm
PIC	~4.9	~130	~1.51	~1.49	0.32	38
I_{max} model	4.93	129.9	1.445	1.66	0.33	
I_{min} model	4.93	129.6	1.445	1.47	0.25	
z_V model	4.90	129.1	1.444	1.47	0.41	
PIC	~4.4	~141	~1.22	~1.84	0.53	31.2
I_{max} model	4.38	140.4	1.227	1.95	0.54	
I_{min} model	4.38	140.3	1.227	1.73	0.45	
z_V model	4.34	139.2	1.227	1.73	0.59	
PIC	~3.7	~153	~1.12	~2.01	0.64	24.1
I_{max} model	3.69	153.2	1.115	2.15	0.66	
I_{min} model	3.69	153.2	1.115	1.91	0.58	
z_V model	3.65	151.6	1.115	1.91	0.70	

The described theory allows to define the MITL impedance which "sees" the reflected wave. This impedance, z_R, is given by

$$z_R = \frac{U_{a1} - U_{a2}}{I_2 - I_1} = 511\frac{\gamma_{a1} - \gamma_{a2}}{I_2 - I_1}, \tag{9}$$

and can be calculated by using the equations presented above. Figure 5 shows the ratio z_R/z_V as a function of z_L/z_1 in I_{max} model for U_{a1} = 5 and 10 MV. For fixed voltage, z_R/z_V depends on z_L/z_1 only, at U_{a1} > 5 MV it exceeds ~0.9 for any impedance of the load $z_L < z_1$.

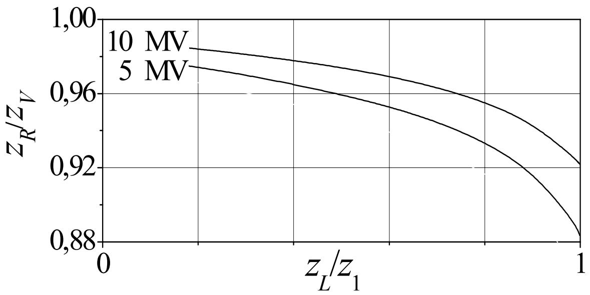

Fig. 5. The impedance z_R in I_{max} model.

5. Conclusion

The presented analytical theory describes the reflection of the Brillouin flow from the load in MITL. The input parameters for this theory are the parameters of the forward going wave, the vacuum impedance of the MITL, and the impedance of the load. It is shown that for both I_{max} and I_{min} models, describing the forward going wave, the predictions of the theory are close to PIC simulations presented in [3]. In conditions of Table 1, the I_{min} model predicts better the re-trapping effect, whereas the I_{max} model predicts better the velocity of the reflected wave.

The theory predicts also that at high enough voltage the reflected wave "sees" the vacuum impedance of the MITL. This allows to use simple approach proposed in [3] to determine the parameters of the reflected wave.

References

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This work was supported in part by the Russian Foundation for Basic Research under the project No. 06-08-01267-a, and by the young scientist's grant of the Siberian Branch of Russian Academy of Sciences.