# Estimation of an Utmost Efficient Potential of Ultrawideband Radiating Systems 

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#### Abstract

In the paper the task of amplitude maximization of an arbitrary antenna nonstationary field was considered. Comparisons of experimental and calculated results for different types of ultrawideband antennas were made.


Different aspects of the task concerning amplitude maximization of an arbitrary antenna nonstationary field at the given moment of time and in the given far-field observation point were considered in the paper both theoretically and experimentally. It is supposed that the maximum linear antenna dimension is such that it is completely located inside the imaginary sphere of the radius $a$. Antenna excitation is realized by a pulse with the limited frequency band and it is supposed that the input energy W is completely radiated by the antenna. It is well known [1] that the antenna field outside such sphere (for monochromatic radiation mode) can be presented as multipole expansions. Coefficients of these expansions are determined by the character of density distributions of electric and magnetic currents in the sphere volume. For this purpose it is convenient to
use [1] the following concepts for the radial components of electric $A_{r}(r, \theta, \varphi)$ and magnetic $F_{r}(r, \theta, \varphi)$ vector potentials that are solely different from zero:

$$
\begin{gathered}
A_{r}(r, \theta, \varphi)=\sum_{n=0}^{\infty} \sum_{m=0}^{n} a_{m n} h_{n}(k r) P_{n}^{m}(\cos \theta) \cos \left(m \varphi+\alpha_{m n}\right), \\
F_{r}(r, \theta, \varphi)=\sum_{n=0}^{\infty} \sum_{m=0}^{n} b_{m n} h_{n}(k r) P_{n}^{m}(\cos \theta) \cos \left(m \varphi+\beta_{m n}\right),
\end{gathered}
$$

where $r, \theta, \varphi$ are the spherical coordinates of the observation point, $h_{n}(k r)$ are Hankel spherical functions of the second type in Debye definition, $k$ is the wave number, $P_{n}^{m}(\cos \theta)$ are Legendre associated functions, coefficients $a_{m n}, b_{m n}$ are the frequency functions not depending on the observation point position, $\alpha_{m n}, \beta_{m n}$ are the constants determining polarization of radiation; time dependence looks like $\exp (j \omega t)$.

Electromagnetic field components that are transverse relative to the radial direction and necessary in the subsequent analysis, being expressed through the potentials $A_{r}(r, \theta, \varphi), F_{r}(r, \theta, \varphi)$, have the following form:

$$
\begin{align*}
& E_{\theta}=\frac{1}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n}\left\{-a_{m n} j Z_{0} h_{n}^{\prime}(k r) \frac{d P_{n}^{m}(\cos \theta)}{d \theta} \cos \left(m \varphi+\alpha_{m n}\right)+b_{m n} \frac{m}{\sin \theta} h_{n}(k r) P_{n}^{m}(\cos \theta) \sin \left(m \varphi+\beta_{m n}\right)\right\},  \tag{1}\\
& E_{\varphi}=\frac{1}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n}\left\{a_{m n} j Z_{0} \frac{m}{\sin \theta} h_{n}^{\prime}(k r) P_{n}^{m}(\cos \theta) \sin \left(m \varphi+\alpha_{m n}\right)+b_{m n} h_{n}(k r) \frac{d P_{n}^{m}(\cos \theta)}{d \theta} \cos \left(m \varphi+\beta_{m n}\right)\right\},  \tag{2}\\
& H_{\theta}=\frac{1}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n}\left\{-b_{m n} \frac{j}{Z_{0}} h_{n}^{\prime}(k r) \frac{d P_{n}^{m}(\cos \theta)}{d \theta} \cos \left(m \varphi+\beta_{m n}\right)-a_{m n} \frac{m}{\sin \theta} h_{n}(k r) P_{n}^{m}(\cos \theta) \sin \left(m \varphi+\alpha_{m n}\right)\right\},  \tag{3}\\
& H_{\varphi}=\frac{1}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n}\left\{b_{m n} \frac{j}{Z_{0}} \frac{m}{\sin \theta} h_{n}^{\prime}(k r) P_{n}^{m}(\cos \theta) \sin \left(m \varphi+\beta_{m n}\right)-a_{m n} h_{n}(k r) \frac{d P_{n}^{m}(\cos \theta)}{d \theta} \cos \left(m \varphi+\alpha_{m n}\right)\right\}, \tag{4}
\end{align*}
$$

where $\mathrm{Z}_{0}$ is the wave impedance of the antenna environment, $h_{n}^{\prime}(k r)$ is the derivative from Hankel spherical function by the complete argument.

Choosing the spherical coordinate system so that the direction $\theta=0$ should coincide with the direction of radiation optimization and taking into account the relations

$$
\left.\frac{d P_{n}^{m}(\cos \theta)}{d \theta}\right|_{\theta=0}=\left.\frac{m P_{n}^{m}(\cos \theta)}{\sin \theta}\right|_{\theta=0}=\left\{\begin{array}{l}
0 \quad n p u m \neq 1 \\
-\frac{n(n+1)}{2} n p u m=1
\end{array}\right.
$$

we come to the conclusion that only the coefficients $a_{1 n}$ and $b_{1 n}$ from (1) - (4) determine maximum value of the field in the direction $\theta=0$. Hence, the coefficients $a_{m n}$ and $b_{m n}$ with $m \neq 1$ determine the energy transported in the directions $\theta \neq 0$. That's why further when maximizing the electric field amplitude at a fixed energy at the antenna input we'll suppose that in (1) - (4) $a_{m n}=b_{m n}=0$ at $m \neq 1$. Without loss of generality we can suppose as well that the field radiated in the direction $\theta=0$ is polarized along the axis

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$x$ related to the spherical coordinate system, so that $\alpha_{1 n}=\pi, \beta_{1 n}=\pi / 2$.

At the given requirements and stated assumptions the component $E_{\theta}(r, \theta, \varphi, \omega)$ in the plane $\varphi=0$ is described by the expression:

$$
\begin{align*}
E_{\theta}(r, \theta, 0, \omega)=\frac{1}{r} \sum_{n=1}^{\infty} & \left\{a_{1 n} j Z_{0} h_{n}^{\prime}(k r) \frac{d P_{n}^{1}(\cos \theta)}{d \theta}+\right.  \tag{5}\\
& \left.+b_{1 n} h_{n}(k r) \frac{1}{\sin \theta} P_{n}^{1}(\cos \theta)\right\} .
\end{align*}
$$

From physical point of view, separate terms of the expansion (5) can be treated as proper waves of a waveguide in the form of free space that satisfy the condition of radiation to infinities and are characterized by availability of a cutoff frequency that allows limiting the number of expansion terms taken into account. Thereby, the problem of rescue from physically unrealizable current distributions resulting in "superdirective" decisions is solved naturally. It should be noted that the number of accountable expansion terms is directly related to the dimensions of the volume occupied by the sources. And namely, the number of the expansion terms $N$ providing the acceptable value of the antenna Q -factor should be summarized. This number can be chosen by three different ways. In case of narrow-band antennas it is usually supposed that $N=\left[\omega_{0} a / c\right]$ where [...] is the integer part of the corresponding number, $\omega_{0}$ is the central frequency of the radiated frequency spectrum. If an antenna radiates in a wide frequency band, then increase of $\omega$ should result in increase of N but so that appearance of a "superdirectivity" mode should be excluded. Such approach exactly is used in Ref. [2]. At the same time, calculations of the field in the far-field zone in the problem of diffraction at a sphere indicate to the necessity of choosing $N=\left[2 \omega_{0} a / c\right]$ in order to provide the $2 \%$-accuracy. However, detailed analysis of results of the works [3, 4] has shown that it is more correctly to choose $N$ according to the following criterion: $N=\left[\omega_{0} a / c+2 \pi\right]$.

Let us assume that in the nonstationary excitation mode an antenna radiates a signal in a limited frequency band characterized by the ratio $2 \Delta \omega / \omega_{0}$. We introduce a designation
$\Omega=\left\{\omega:-\omega_{0}-\Delta \omega<\omega<-\omega_{0}+\Delta \omega ; \omega_{0}-\Delta \omega<\omega<\omega_{0}+\Delta \omega\right\}$. Then electric field excited by this antenna in the region $r>a$ can be written as

$$
E_{\theta}(r, \theta, 0, t)=\frac{1}{2 \pi} \int_{\Omega} E_{\theta}(r, \theta, 0, \omega) e^{j \omega t} d \omega
$$

where integration is made both by positive and negative frequencies.

Full radiated energy is determined by the expression

$$
W=\frac{1}{2 \pi} \int_{\Omega} \oint_{S}\left[\mathbf{E}(r, \theta, \varphi, \omega), \mathbf{H}^{*}(r, \theta, \varphi, \omega)\right] \mathbf{n} d s d \omega
$$

where $S$ - is the sphere surface of the radius $a, \mathbf{n}$ is the external normal to this surface.

Using (1) - (4) and orthogonality relations for trigonometric functions and Legendre associated functions $P_{n}^{1}(\cos \theta)$ and making integration by the surface $S$, we obtain

$$
\begin{array}{r}
W=\int_{\Omega}\left\{\sum _ { n = 1 } ^ { N ( \omega ) } \frac { n ^ { 2 } ( n + 1 ) ^ { 2 } } { 2 n + 1 } \left[j\left|a_{1 n}\right|^{2} Z_{0} h_{n}^{\prime}(k a) h_{n}^{*}(k a)-\right.\right.  \tag{6}\\
\left.\left.-\frac{1}{Z_{0}}\left|b_{1 n}\right|^{2} h_{n}(k a) h_{n}^{\prime *}(k a)\right]\right\} d \omega .
\end{array}
$$

Let us introduce new coefficients into consideration

$$
A_{n}=j^{n+1} Z_{0} n(n+1) a_{1 n}, B_{n}=j^{n+1} n(n+1) b_{1 n} .
$$

Then expressions (5) at $\theta=0, k r \rightarrow \infty$ and (6) will be symmetric relative to the coefficients $A_{n}$ and $B_{n}$ and that's why maximum $\left|E_{\theta}(r, 0,0, \omega)\right|$ will be achieved at the condition of $A_{n}=B_{n}$. If this condition is fulfilled and $k r \rightarrow \infty$, then the expression (5) takes the following form:

$$
E_{\theta}(r, \theta, 0, \omega) \approx \frac{e^{-j k r}}{r} \sum_{n=1}^{N(\omega)} \frac{A_{n}}{n(n+1)} L_{n}(\theta),
$$

where $L_{n}(\theta)=-\frac{d P_{n}^{1}(\cos \theta)}{d \theta}-\frac{P_{n}^{1}(\cos \theta)}{\sin \theta} \quad$ and
$L_{n}(0)=n(n+1)$.
According to this, in the time domain we'll obtain

$$
\begin{equation*}
E_{\theta}(r, \theta, 0, \tau)=\frac{1}{2 \pi r} \int_{\Omega} \sum_{n=1}^{N(\omega)} \frac{A_{n}}{n(n+1)} L_{n}(\theta) e^{j \omega \tau} d \omega \tag{7}
\end{equation*}
$$

where $\tau=t-r / c$.
Expression (6) for radiated energy after using wronskian relation for the functions $h_{n}(k a)$ and $h_{n}^{*}(k a)$ is written as follows

$$
\begin{equation*}
W=\frac{2}{Z_{0}} \int_{\Omega}^{N(\omega)} \sum_{n=1}^{N(\omega)} \frac{\left|A_{n}\right|^{2}}{2 n+1} d \omega \tag{8}
\end{equation*}
$$

Let us optimize the amplitude of the field radiated in the direction $\theta=0$ at the time $\tau=0$. It is convenient to determine a functional for the electric field amplitude in the far-field zone in the direction $\theta=0$ and at $\tau=0$ by writing (7) in the scalar product terms in the following way [2]

$$
E_{\theta}(r, 0,0,0)=\left([A]^{t},[F]\right),
$$

where $[A]$ is the column vector composed of $A_{n},[F]$ is the column vector with $F_{n}=1 / 2 \pi r$, and symbol $t$ denotes transposition. The scalar product is determined as

$$
\left([A]^{t},[B]\right)=\int_{\Omega}[A]^{t}[B]^{*} d \omega .
$$

Expression for the energy (8) can be presented in the form of [2]

$$
W=\left([A]^{t},[H][A]\right),
$$

where $[H]$ is the diagonal square matrix with elements $\quad H_{m n}=\frac{2 \delta_{m n}}{Z_{0}(2 n+1)}, \delta_{m n}$ is the Kronecker's symbol.

We maximize the value of the field determined by the expression (7) at the given energy value $W$. This is achieved if the equation is fulfilled [5]

$$
\nabla\left(E_{\theta}(r, 0,0,0)-\lambda W\right)=0
$$

An optimum solution is found using the methods described in detail in Ref. [5]. The result is the expression for the coefficients

$$
\begin{equation*}
A_{n}=\frac{Z_{0}(2 n+1)}{8 \pi \lambda r}, \tag{9}
\end{equation*}
$$

where $\lambda$ is Legendre multiplier chosen so that the calculation by the formula (8) should result in obtaining the given value of $W$.

The calculation algorithm for the efficient potential (product of $r$ by $E_{\theta}$ ) of a system is reduced to the following. After substituting (9) into (8) we find Legendre multiplier $\lambda$. As a result, the values of the optimum coefficients $A_{n}$ turn out to be completely determined

$$
A_{n}=\sqrt{Z_{0} W} \frac{(2 n+1)}{\sqrt{2 \int_{\Omega}\left[\sum_{n=1}^{N(\omega)}(2 n+1)\right] d \omega}}
$$

Their substitution into the expression (7) results in the final calculation formula

$$
\begin{equation*}
r E_{\theta}(r, 0,0,0)=\frac{\sqrt{Z_{0} W}}{2 \sqrt{2} \pi} \sqrt{\left.\int_{\Omega}^{\left[\sum_{n=1}^{N(\omega)}\right.}(2 n+1)\right] d \omega} . \tag{10}
\end{equation*}
$$

Initially, we compare the efficient potentials evaluated by the formula (10) at the value of $N=\left[\omega_{0} a / c+2 \pi\right]$ that we have chosen and the value of $N=\left[\omega_{0} a / c\right]$ used in Ref. [2]. Fig. 1 presents the ratio of the efficient potentials $r E_{2 \pi} / r E_{0}$ for radiation with the frequency bands of $10 \%$ (curve 1) and $150 \%$ (curve 2).


Fig.1. Ratio of efficient potentials versus antenna electric dimension for radiators with the frequency bands of $10 \%$ (1) and $150 \%$ (2).

It is seen that for electrically small antennas this value can exceed 10 and it tends to 1 for larger antennas. Thus, application of $N$ that we have chosen
allows estimating the utmost values of the efficient potential of arbitrary antennas.


Fig.2. Measured efficient potential relative to the utmost one (1) and the sphere radius (2) versus the number of combined antennas in the array.

It is interesting to compare antennas widely used for radiation of high-power ultrawideband (UWB) pulses by the chosen criterion. Here belong IRA [6], TEM [7] and combined antennas (CA) [8]. The following values were estimated by the information presented in these papers: energy and spectrum of radiation for IRA as well as energy and spectrum of voltage pulse at TEM and CA inputs. In the latter case, the antenna efficiency by energy was supposed to be equal to $100 \%$. Frequency band was evaluated by the level of -10 dB , and $\omega_{0}$ was chosen as the average value in this band. Spectrum width for the antennas under investigation was in the limits of $100-$ $200 \%$. For the comparative analysis, estimation of $r E_{\theta}=r E_{2 \pi}$ for different radiators was made and efficiency coefficient values were obtained as ratios of the experimentally measured potential to the utmost one $k=r E_{\text {exp }} / r E_{\theta}$. The coefficient $k$ for IRA, TEM and CA equals, respectively, to $0.5,0.35$, and 0.26 . For a 16 -element array of the combined antennas excited from one generator [8] $k=0.58$. As it follows from the results presented in Fig. 2, for a 100element array $k=0.75$ that is by a factor of $\sim 1.5$ higher than for IRA. Estimations show that for CA and CA-based arrays the ratio $r E_{\theta} / V$ where $V$ is the radiator volume is by an order of magnitude higher than for other radiators.

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